A Real Options Theory:

Quantifying and valuing the possibility of a lease renewal

university of groningen

Master Thesis
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August 2012

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Preface

For my graduation I wrote my master thesis at the Valuation Advisory department of CBRE Netherlands in Amsterdam. My research is derived from a particular question where CBRE was keen to get an answer on. They wanted to know if it is possible to forecast the possibility that a tenant would renew his contract.

The results of my thesis should be able to be of use for CBRE when appraising a building with a tenant or multiple tenants. Because I am a rather rational person and like to think about abstract and quantitative problems, the real option theory quickly came into my mind and in that of my graduation mentor. This is a theory that is derived from the financial option theory. To understand what the real option theory was, I dug into the financial option theory and I must say this was very interesting and very instructive. As this is quite an econometric subject, I received great help from an econometrics professor of the Erasmus University Rotterdam.

The result of my thesis is a theory that is 'new' in real estate. Where the real estate market and professionals always leaned on experience, historical date and intuition, the base of the real option theory is rational, forecasting and abstract. I do not know if the real estate market is ready for this way of thinking. But it sure gives a thought about how it could be done.

I would like to thank everyone who has contributed in any way to my master thesis. In particular, I would like to thank my graduation mentor, Drs. Arthur Marquard from the University of Groningen and the Amsterdam School of Real Estate, Dr. Ronald Huisman, Econometrics professor at the Erasmus University Rotterdam and the Amsterdam School of Real Estate and Walter de Geus and Kees van Vilsteren from CBRE, they gave me the opportunity to write my thesis at CBRE. And last but not least, my mother, who made it possible for me to attend college.

With pleasure, I am looking back on an interesting and instructive period at the University of Groningen as well as at CBRE.

They say that the period as a student is the best of your life. A period where everything is possible and where there are almost no limitations to your freedom. Looking back on that time of my life, the only thing I can say is that they are right. Herewith, it is over.

Amsterdam, 8th of august 2012

Index

Abstract

Currently, the possibility that a tenant will renew his contract, i.e. the risk that a tenant will not renew his contract, is processed in the required return. This is the way the traditional methods process this possibility. It is not further specified or quantified in any way.

The banks, investors and other financial institutions are demanding a better explanation and foundation of this possibility. Therefore, CBRE Netherlands asked to investigate if it is possible to quantify this risk. In this research, at the same time a value is assigned to this risk.

As the traditional methods are not capable of quantifying and valuing risk, an alternative method has been used. The method that is found and used is the real option theory, a theory based on the financial option theory. This theory makes it possible to process possibilities and uncertainties in the valuation of real estate investments. An existing model, the Black – Scholes model, has been altered and used to quantify the exercise possibility of a rental lease renewal, at the same time this model determines a value to this possibility.

In this research, several simulations of cases with different scenarios are shown to see what kind of influence changes in input parameters have on the exercise possibility and the value of the option. The main factor which determines the exercise possibility is the combination of volatility and time till exercise.

However, the method and the results are very rational and abstract. To use this method and to interpret the results, in a way that they are useful in practice, further research is essential. A follow- up research is needed to test the theory and results in practice.

If the theory and results are in accordance with the practice, the model and theory from this research can give a good indication about the renewal possibility of the future.

1 Introduction

This chapter gives an outline of the research. The motivation, the scientific and social relevance of this master thesis, the problem statement is defined as well as the objective of the research. Also, the methodology is explained and why this method has been chosen. Finally, this chapter ends with an outline of the structure of the research.

1.1 Motivation

Any form of investment brings along certain risks. For an investor in real estate, one of the biggest risks is loss of (rental) income, i.e. vacancy of the property. Or, the possibility that a tenant will not renew his contract. Since the credit crunch banks, investors and other financial institutions are more careful to lend out money for real estate projects. If they lend out money, they want to be sure that the investment has a good rate of return. Therefore, if they lend out money to finance real estate developments, the risk analysis they want must be as thorough as possible. If these institutions want an independent and objective appraisal of their investments, the large international real estate consultancy firms are usually the parties that perform these appraisals for them. Because the required return, also called the discount rate, is the reflection of the risk of the underlying asset that is to be appraised, the required return must be determined in an objective and thorough way to get the right reflection of risk in the underlying asset.

When appraising a real estate project, risk is brought under in the required return. The risk that a tenant will not renew his contract is also processed in the required return. The required return consists of two main layers, the risk free interest rate and a risk premium. The higher the risk, the higher the required return. Also, the risk premium arises from comparisons with other required returns in the market and somewhat subjective adjustments (Van Gool, 2007). But in the real estate market, risks are determined in a rather intuitive way (Hishamuddin, 2000). Because of the credit crunch and the growing uncertainty if an investment will deliver a good rate of return, banks, investors and other financial institutions are not satisfied anymore with an appraisal where the required return is based upon subjective and intuitive methods.

With this in mind, the banks, investors and other financial institutions are asking the large international real estate consultancy firms for valuation reports that are more comprehensive, thorough and detailed and must have a better research foundation than before. Currently, it is suggested that these companies have not quantified each risk in a specific way. Instead, they process the risks in an intuitive way, based on specific experience and historical data. This intuitive method is prone to subjective influences.

Because the Valuation Advisory department at the CBRE office in Amsterdam is getting a lot more questions and requests from the banks, investors and other financial institutions about the risk of vacancy, the question to quantify this possibility derives from them. They would like to see this risk quantified in a scientific way. So, they have a solid answer or indication that is based on scientific research. Therefore, this scientific research is made on behalf of this specific question that comes from the Valuation Advisory department of CBRE Amsterdam.

As this is a question that comes from the Valuation Advisory department, the value of this risk is also an interesting aspect to investigate. Therefore, this research will not only quantify the possibility that a tenant will renew his lease contract, but also assign a value to this risk. Appraising risk or possibilities is not possible to do with the traditional methods, the gross initial yield method, the net initial yield method or the discounted cash flow method (Nederhorst, 2009). So, to appraise risk or possibilities, another method to appraise is used. Because this method is rather new and unknown in real estate, this research is giving a look into this new method. Also this research will argue how this method can contribute in appraising risk and possibilities in real estate.

1.2 Relevance

The scientific relevance of this research is present. There are a lot of researches present that are giving reasons why a tenant would renew his contract or not and which factors are important for a tenant to renew. But, there is a lack of scientific or theoretical research present to determine and forecast the possibility of a lease renewal. Because it is such an important risk for investors and financial institutions it is odd that this is an underexposed subject in real estate to do this in a quantitative way. It is important to determine risk in an objective and rational way. This research is hoping to contribute to lay a basis for a scientific, rational and quantitative way to determine risk and assign a value to this risk.

The real estate market has gained relative bad media attention in the last few years. Several cases of fraud in real estate were in the spotlights of the media. The real estate market still has not got the image that it would like to have. The market is not seen as a transparent and healthy market. This is in contrast with changes in society these days. In social terms, transparency and integrity are becoming more important every day in every branch of society. Seen from a social point of view, this research is improving the transparency of the real estate market. To determine risk in a quantitative and complete rational way, the intuitive aspect of real estate, and therefore subjective, is partly removed.

Also, this research shows a new way of thinking. A new perspective and point of view about appraising in real estate. Where the real estate professionals today most of the time use the traditional methods, this research highlights the possibilities of a new method. Because the traditional methods are static and rigid, this new method is capable to appraise risk and possibilities, where the traditional methods are not.

1.3 The research

The problem statement of this research is:

- *The possibility that a tenant will renew his lease contract is currently based upon intuitive aspects and is not quantified in a scientific and objective manner.*

The objective of this research is:

- *To gain insight in the manner how the possibility that a tenant will renew his lease contract is quantified.*

In this research, the main questions and several sub questions are answered. As this research is written for the Valuation Advisory department of CBRE Amsterdam, the value of this possibility is also of great importance. Therefore, this research has two main questions that need to be answered.

The main questions of this research are:

- *In which way is it possible to quantify the possibility of a lease renewal?*

- *To what extent is it possible to assign a value to this possibility?*

To solve the problem that is stated and to answer the main question, several sub questions must be answered. This should and must create an answer to the main question and therefore also a solution for the problem statement.

The sub questions in this research are:

- *How is the possibility that a tenant will renew his lease contract now determined?*

- *In what way is this possibility currently taken into account in appraising real estate?*

- *Can the possibility that a tenant will renew his contract be determined by means of the traditional methods?*

- *How is it possible to apply the (real) option theory to determine the possibility that a tenant will renew his contract?*

- *Is it possible to translate this theory to real estate lease contracts?*

- *Which factor(s) determine (s) the exercise possibility and the value of the option?*

- *In what way can the real option theory for real estate lease contracts be applied in practice?*

1.4 Methodology

The methodology of the thesis is solely theoretical. An empirical research is not included in this research. The reason for a purely theoretical research is because the theory that is discussed, the real options theory, is rather new. Because there is limited literature about the real option theory, and especially related to real estate lease contracts, it is important to lay a theoretical foundation that investigates the applicability of the theory to the investigated subject.

To apply, or to test, the theory in practice, a well-defined theoretical base must be made. In this research, this is done by means of literature review. Many books, scientific journal articles used to lay the theoretical foundation of the subject. Also, several informal discussions with professors in real estate and econometrics have taken place to discuss the matter.

The combination of an extensive literature review and the informal discussions with real estate en econometric experts must be sufficient to produce theoretical basis for potential further research. So, the emphasis of this research is on literature review and theoretical research, not on empirical research.

The simulation of several cases in Chapter 6 of this research is a simulation that is based upon theoretical assumptions, not empirical.

1.5 Structure

The structure of the thesis is as follows: In chapter 2, the current methods that are used to appraise real estate are discussed. The possibility of the renewal of a lease contract is a risk and therefore, in case of the traditional methods, a part of the risk premium that is processed in the required return. So, the required return and the risk premium are also discussed.

Chapter 3 will explain the option theory that is used in finance. Here is explained what an option is, what kind of options there are and how an option is given a value. This chapter explains the way of thinking and the foundations of the new method that is introduced in this research.

In chapter 4, the real option theory is discussed. This is the step towards the implementation of the option theory to real estate. Here is explained how the real option theory can be used in the real estate market and how this must be done.

Next is chapter 5. In this chapter the translation of the real option theory to real estate lease contracts is made. The model that is used for this research is explained and the most important characteristics are discussed.

In chapter 6, a simulation of some case examples are shown to explain the model that is used. This simulation shows what the influence is on the possibility of a lease contract renewal and the value of this possibility, when some parameters are changing.

Finally in chapter 7, a conclusion is drawn about the application of the real option theory, relating to real estate lease contracts. Also, some recommendations are made about how to interpret the outcomes of the model and how to use this in practice.

2 Traditional methods

This chapter discusses the current methods that are used to appraise real estate. As the renewal possibility of a lease contract is processed in the required return, the most used appraisal methods that use a required return also will be discussed. The required return consists of the risk free interest rate and a risk premium. Several researches about this risk premium will be discussed. It seems that there is not a consistent and specific method that is applied to determine this risk premium. It is quite a grey area. This aspect does not improve the uniformity and transparency of the appraisal business.

2.1 Valuation Methods

In general there are three methods to appraise real estate (Lusht, 2001). These are:

- Comparative method
- Cost approach method
- Income approach method

This research is about the quantification of a certain risk that currently is put in the risk premium. The comparative and the cost approach method do not use a required yield and therefore no risk premium. These methods will therefore not be discussed further in this research. However, the income approach is using a required yield and therefore a risk premium. This method is divided in three different sub methods, namely:

- Gross Initial Yield method
- Net Initial Yield method
- Discounted Cash Flow method

2.1.1 Gross initial yield and net yield method

The gross initial yield method is one of the methods that is applied the most by appraisers. It is a method that is quick and easy to use. It is a relative simple method, as it uses a limited numbers of variables. The calculation model implies that the market value equals the gross rental income divided by the gross initial yield.

The gross initial yield is according to Osinga (2000) the most important indicator of the mood on the real estate market. The gross initial yield is compared with other yields of investment assets like government bonds or shares. The gross initial yield that is compared with another gross initial yield of another object should represent elements of the degree of risk, potential growth of value and the general yield of the market. In this point of view, the gross initial yield can be considered as an initial yield that is adjusted for risk.

The formula of the gross initial yield method to determine the value of an object is as follows:

$$\textit{Value} = \frac{\textbf{Annual gross rental income}}{\textbf{Gross Initial Yield}}$$

The annual gross rental income means the total rental income without taking any expenses into account. First, the rental income is determined by means of the comparative method. Of course, it is possible that the real rental income (contract lease rent) is higher or lower than the market rental value (Van Hulst, 2004).

In branch magazines, like Vastgoedmarkt and PropertyNL, transactions are published frequently. In these published transactions the gross initial yield is often mentioned. The market data about the height of the gross initial yield is therefore widely available. Because of these publications a comparison with the 'market gross initial yield', and therefore a negative or positive correction, can be made to determine the gross initial yield (Van Hulst, 2004) which can be applied in the method.

The gross initial yield method has as biggest advantage that it can be applied relatively simple (Van Gool et al, 2007). On the other hand the GIY method has also several disadvantages according to Van Hulst (2004) and Van Gool et al (2007). These are:

- If there is little market evidence and/or with objects where 'permanent' vacancy is present this method is difficult to apply. The trading liquidity of these objects is low, because of this, relevant and recent market data is missing. The risk of a too low gross initial yield, thus a higher value, is more present. If vacancy occurs or increases, appraisers apply a so called gross initial yield with a 'vacancy discount'.
- In the determined gross initial yield hidden assumptions can be present.
- The GIY method is unusable when the cash flows in the future are expected to be very volatile. This is because the method is based on a perpetual cash flow where, in theory, there is no difference between the growth rate of the rental income and of the other cash flows, for instance the exit value.
- The gross rental income that is used is a snapshot in time.
- There is no insight into the costs of exploitation.

The use of the net initial yield method has been increasing in the last few decades. Appraisals for the IPD index in the Netherlands must be done by means of this method. The only other permitted method is the discounted cash flow method. This method was mostly used by financiers because the owner's charges were made visible whereby the net cash flow available for interest and repayment is well mapped out. Because of this, the net initial yield method is more accurate than the gross initial yield method. On the other hand critics say that the use of exploitation costs will give a higher

margin of error. This because a few factors, which determine the exploitation costs, are difficult to define. These factors are for example the maintenance costs. Questions and/or ambiguities can arise about which parts should be taken into account and which ones should be left out, like renovations and major maintenance (Van Hulst, 2004).
The formula of the net initial yield method to determine the value of an object is as follows:

$$\textit{Value} = \frac{\textit{Annual Gross rental income} - \textit{Exploitation Costs}}{\textit{Net Initial Yield}}$$

2.1.2 Discounted Cash Flow method

The discounted cash flow method means that during a review period all income and expenses are put back in time. Subsequently these numbers are netted and per year the obtained number is discounted to the present. Then the numbers of all years are added together. This method was already widely used in financial calculations. The use of this method in real estate found its origin in the United States during the 1950's. In this period financing with borrowed capital became more common. Previously the base of appraising real estate was found within the physical characteristics of the object. The base gradually moved towards the financial characteristics of the real estate. The discounted cash flow method became more popular as a result of the realization that the prices of real estate became more stable over the years, and even increased and that the value of the objects were determined by the increasing annual rental income. The shift of real estate to an investment instead of an object for own use was also of great influence (Van Hulst, 2004). The traditional comparative approach became, logically, more difficult to use in this situation.

The discounted cash flow method is mostly used by investors to calculate the market value and the investment value (Van Gool, 2007). The internal rate of return can also be calculated, this is possible by entering the current value or the purchase price. Subsequently, the internal rate of return can be determined as a resultant (Van Hulst, 2004). The difference in value that occurs between the investment value and the market value is caused by the interest rate that is used to calculate the present value. The investment value is discounted by a subjective required return of interest. The market value is determined by a certified appraiser, and therefore is being valued by means of an objective discount rate. Therefore, it is of great importance that the appraiser must determine the discount rate in the most objective way possible, but must also take the actions of the investor into account to appraise in accordance to market terms (Van Hulst, 2004). According to Van Hulst (2004) the discounted cash flow method has several advantages and disadvantages. These are:

Advantages

- More transparency, in comparison to other methods. With the discounted cash flow method there is a larger insight in cash flows.
- The method is focused on the future; therefore the value is based upon the future.

- The method is well applicable with volatile cash flows (investing, to sell off individual units).
- Few hidden assumptions possible.
- The method is consistent with the method (institutional) investors use and therefore communication is (more) clear.
- Good correspondence with the new IFRS regulations.
- Benchmarking for the purpose of the IPD.
- The appraiser is forced to investigate several parameters.
- The possibility to calculate different scenarios.

Disadvantages

- The large number of parameters that needs to be determined.
- To estimate macro-economic variables.
- To determine the exit value.
- To determine the required yield/discount rate.
- The lack of direct market evidence.

2.2 Breakdown of the required return

The gross initial yield, net initial yield or the discount rate, are all synonyms for the required return. The required return is a reflection of the risk of an investment. The investor wants to be compensated for the risk he is exposing himself to (Lusht, 2001). The required return consists of a risk free interest rate and a risk premium. 10 year government bonds are often used to determine the risk free interest rate (D'Argensio and Laurin, 2008). The extra risk that an investment entails in comparison to a 10 year government bonds is represented by the risk premium. Combining these displays of risk, a representation of the required return is given. According to Robijn (2011), the risk premium consists of the conventional premium for real estate (approximately 2%). Next to the conventional premium a further premium or discount is based upon market feeling, the opportunity cost of capital, interest on loan, geographical factors, specific object factors and sectorial factors. Langens (2002) also has done research concerning the breakdown of the required return. The risk premium, according to him, is determined by inflation, the condition of the real estate market, the condition of the investors market, the quality and the location of the object, the sector the object is in, the possible growth of rent and value, the length of the lease contract, the quality of the tenants, the possibility of vacancy and the type of ownership. Lusht (2001) uses the following characteristics of which a risk premium consists: the risk for investing in real estate, the risk of unexpected inflation and the risk that is linked with the illiquidity of real estate. The risk concerning the quality of the tenant, the composition of the lease contract, the characteristics and quality of the object, the possibility of vacancy etc. can be interpreted as the risk involved when investing in real estate. The 'Vereniging van Nederlandse Gemeenten' (Association of Dutch Municipalities) has in its appraisal guideline (Taxatiewijzer Huurwaardekapitaliesatiefactor) a definition of its own of the required return (Vereniging van Nederlandse Gemeenten, 2011). The factors that make up the risk premium according to this definition are the risks that are related to the possibility of vacancy, the instability of the tenant, the type of real estate, the location and unknown risks of the economy. Considering the several reports concerning the breakdown of the required return discussed above, there is some consensus on the factors determining the risk premium. The different mentioned factors are in general quite similar.

The way to determine the risk premium can differ. Van Gool et al (2007) confirm this by saying that there are three different ways to determine the height of the risk premium. The 'natte vinger' method, this is a method solely based on the experience, market conditions and (subjective) opinion of the appraiser or investor. The second method is the one that is based on the historical required return-risk ratios. The last method determines the risk premium, beta, by means of the Capital Pricing Model (CAPM).

In practice, appraisers and investors in the Netherlands predominantly use the so-called 'natte vinger' method (Van Gool, 2007). Appraisers and investors are mainly looking to compare the returns that are being determined in the real estate market. This is to prevent that they are pricing themselves out of the market. In this method the risk free interest rate is based upon the return on the effective returns of long term government bonds. Also a difference is made between the different investment categories. The risk premium per category differs and is therefore determined differently. According to Van Gool (2007) this method is good at first sight, but the different risk premiums are determined on a very subjective basis. It seems that the subjective opinion of the appraisers and investors has an important and significant influence on the height and determination of the risk premium and therefore on the required return.

The method that is based upon the historical risk-return ratios, just as the name does suspect, determines the risk premium by means of the historical risk-return ratios per investment category. With help of the efficient frontier it is possible for the investor or appraiser to choose the optimal ratio between risk and return. However the past does not give any guarantees for the future and therefore this method is not applied that much by investors and appraisers (Van Gool, 2007). The series of historical risk-return ratios are being published by Troostwijk and the IPD. However the series of the IPD are based on achieved returns and according to research by Van Hulst (2004) these series are less useful, this is because it concerns an input variable of the calculation model. The series of the IPD are also, for a large part, based upon appraisals and not on transactions.

The last method is the Capital Pricing Model (CAPM). Risk is displayed here as the beta. The beta is the relation between the systematic risk of the investment (diversifiable risk) and the risk of the market as a whole (non-diversifiable risk). With this, the correlation between both returns is taken into account. There are several ways to determine the beta. In principle this can be done on the basis of historical data on returns, risks and correlations. In this model the market is seen as perfect and completely rational. However, according to Van Gool et al (2007), the real estate market is far from perfect and rational. Specific risks that are connected to objects cannot simply be diversified away. Because of this the real risk will always be higher than the model assumes it to be. Often historical data are used to determine the beta that is missing. These are not always consistent.

2.3 Risk premium

At first sight, the eventual determination of the required return seems to be well thought over. But in practice, the determination of the risk premium is often determined through random and subjective decisions. According to Van Hulst (2004) the elements that together compose the risk premium are still a grey area. Van Hulst (2004) also argues that every self-respecting appraiser has developed his or her own sense of feeling of the market and his or her own methodology in appraising. In general a high risk is linked to a low quality and a low risk to a high quality. The question remains, which aspects determine the quality of an object. According to research done by Van Hulst (2004) it seems that a lot of elements that together form the risk premium are object related elements. These elements are among others: the financial quality of the tenant, the duration of the lease contract, the flexibility of the object, the location, the accessibility, the demand and supply of real estate in the area, etc.

Langens (2002) also raises the question which elements determine the risk premium. From his research it can be concluded that in contrast to the risk free interest rate, the risk premium is not specified and determined in a uniform way and according to a standard method. The height of these risk premiums are quantified on a sense of feeling and experience. In his research it becomes clear that estimations of vacancy are taken into account in the future cash flows. Van Hulst (2004) states in his research that some premiums are being used by most appraisers in the determination of the whole risk premium. These are: a general premium for investing in real estate, a sector premium and a premium for the specific object. With all three premiums a clear and well defined motivated underpinning is missing of how the height of these premiums is determined. The bandwidth that is present in the different premiums and the absence of a clear and well defined motivated underpinning suggest that a uniform system is missing. On the other hand, it can be suggested that this is the key to determining the required return. According to Van Hulst (2004), appraisers do not want to reveal their methods to others.

Osinga (2000) has done research about the possibility of determining a risk premium on the theory of financial markets. If so, a required return that is in line with the market will arise. This will make it possible to appraise in an objective and independent way. The current method of determining a required return, and risk premium, of investors and appraisers is, according to Osinga (2000), a rather subjective one. Therefore, the outcome of the calculation, the value, is also considered subjective according to him. The purpose of his research is to obtain an objective method to determine the risk premium. He also advocates using the discounted cash flow method instead or next to the gross initial yield method. The reason for this is that if two methods are used separately for an appraisal and if the outcome of the value is equal or approximately the same, the possibility of a correct and objective value is larger. At this moment the gross initial yield is the most used method. On the basis of the investigated data in his research he claims that his findings are not suitable to determine a required return that is in line with the market. However, he concludes that when determining the risk premium, the growth of the economy and the demand for real estate must be taken into account. How to build up the risk premium and how to quantify this is not further answered in his research.

Kruijt (1994) states in his research that the risk premium that is put into the required return, is related to the expected growth and a constant percentage of exploitation. The expected growth percentages are linked to the current inflation and therefore it is possible that these percentages can fluctuate significantly in time. According to his research there is constant overestimation and underestimation of the future growth of income and value. These over- and underestimations are being influenced by cyclical economic fluctuations. He states that the state of the real estate market is similar to the fluctuations that are occurring with the real interest rates. The risk premiums are very sensitive to this. The risk premium is linked negatively with the real interest rates and positively with inflation.

It can be concluded that there is some form of consensus on the elements a required return should consist of. However, a part of the required return is based upon unclear and partially subjective motivations to determine the height of the risk premium, as used by the appraisers and investors are not clear. Taken this into consideration the determination of the value of the object is partially based upon unclear and subjective motivations, thus the value is possibly not objective. All of this does not benefit the transparency and uniformity of the appraisal branch in the real estate market. The larger (international) real estate consultancy companies are following the rules of the international branch organization Royal Institute of Chartered Surveyors (RICS). This organization is well known for the RICS Red Book, the first standardized appraisal guideline. An important part of this guideline is objectivity and transparency (RICS, 2012). This organization advocates the highest standards in moral guidelines for the real estate market and in the service of delivering unbiased, impartial and neutral appraisal advices. Therefore, the objectivity and transparency that is promoted by the branch organization does not reflect the methods and motivations that are currently being used to determine the risk premiums, thus the appraisal and value.

Through quantifying the different risk premiums it is possible to obtain a higher objectivity. This is because the emotional aspect that is currently present within the subjective motivation and determination of the risk premiums will disappear. The transparency will also increase if the risk premiums will be quantified because the motivations and assumptions will be made clear. The result of this process of quantifying the risk premium is that the motivation and objectivity of the value will increase and therefore a more founded and objective appraisal advice will be given.

2.4 Conclusion

It has become clear that the risk that a tenant will not renew his lease contract is currently processed in the required return that is used in the traditional methods that the appraisers use. The required return used, consists of two main layers, the risk free interest rate and a risk premium. Currently there is no consistency and unified way to determine this risk premium. In real estate risk is not quantified by means of scientific research. Risk is based upon and determined by experience, subjective and intuitive ways. With the traditional methods it is also not possible to assign a value to this risk. The methods that currently are used by the appraisers are static ones. This means that uncertainty has a normal distribution (Trigeorgis, 1999). It is also not possible to determine and appraise these uncertainties. However, in this research the possibility of lease renewal and assigning a value to this possibility is investigated.

When taking a look at a renewal option, one thing can be noticed. An option to renew a contract is a right to do this, not an obligation. This is a characteristic that it shares with a financial option. So, like an option in finance this possibility must have some value. There are methods to value such a financial option and at the same time to determine the exercise possibility. To apply this method, the option theory, to real estate, we first need to understand the fundamentals and philosophy of the financial option theory. In the next chapter this theory is discussed.

3 Option Theory

If someone buys an option he buys the right to engage in that transaction. The seller incurs the corresponding obligation to fulfill the transaction. An option which conveys the right to buy something at a specific price is a call-option. If a tenant wants a renewal option in a contract, in a certain way this can be regarded as a call-option. But what is the value of this option to renew the lease contract? In the financial world there are several methods to appraise an option.

In real estate this is a rather unknown area. In this chapter, the option theory will be discussed. To appraise an option in real estate the real-option theory must be used. This theory is derived from the original financial option theory, but the parameters are adjusted to real estate. To explain how the real option theory can be applied in the matter concerning the renewal of a lease contract and how to value the renewal option, we first must discuss how the fundamentals of the option theory work, namely the financial options.

3.1 Financial options

The most well-known option is the option used in finance. Geltner et al (2007) define an option as follows:

An option is the right ***without obligation*** *to obtain something of value upon the payment or giving up of something else.*

The person that has that right is referred to as the owner or holder of the option. The asset that is obtained by exercising the option is known as the underlying asset. That what is given up is referred to as the exercise price of the option. The holder of the option has the right to exercise the option or not to exercise the option

There are two basic types of options. A call option and a put option. A call option gives the holder/buyer of the option the right to buy an asset by a certain date for a certain price. A put option is the opposite. This gives the holder/buyer the right to sell an asset by a certain date for a certain price. The certain date that is specified in the contract is known as the expiration date or the maturity date. The certain price that is specified in the contract is known as the exercise price or the strike price (Hull, 2010). The renewal of a contract is a call option, because it gives the tenant the option to renew (buy) the contract by a certain date for a certain price. Therefore all the assumptions, factors, explanations, etc. will be focused on, and written from the point of view of a call option.

Two sides are present in every option contract. On the one side is an investor, who has taken the long position (he has bought the option). On the other, there is another investor, who has taken a short position (he has sold or written the option). The writer of an option receives his cash up front. His potential liabilities come later. The profit or loss for the writer is the reverse of that for the purchase of the option (Hull, 2010).

Options can be classified in three other types, American, European or Bermuda options. This has nothing to do with the geographical location of the option. The difference between these three is in the possibility of the time of exercising the option. American options can be exercised at any time up to the expiration date. European options can only be exercised on the expiration date itself. Bermuda options are options where the option holder can exercise the option at several predetermined dates (Vlek et al, 2009). American options are the most common ones to be traded on the exchanges. On the other hand, European options are easier to analyze (Hull, 2010). In both cases, the exercise is irreversible. In the act of exercising the option, the option itself is thereby given up. An option can only be exercised once (Geltner et al, 2007).

In this research a lease contract will be discussed and this contract can be seen as a European option. A lease contract, for say five years, will expire five years from now. This term is fixed and the renewal contract (the option) can only be exercised at the specific date five years from now, when the contract expires. The new contract will only be valid from the expiration date, if renewal is the case. This is the same with a European option where it is only possible to exercise the option at expiration date.

3.2 Valuation of options

The option value can be determined by using a range of quantitative techniques that are based on the concept of risk neutral pricing and the use of stochastic calculus. According to Reilly and Brown (2003) the most basic model for determining the price of an option is the Black-Scholes model. But there are also more sophisticated models that are used to model the volatility smile. These models are implemented using a variety of numerical techniques (Reilly and Brown, 2003).

According to Hull (2010), there are six factors that in general affect the price of a stock option:

- The current stock price, S_0
- The strike price of the option, K
- The time to expiration, T
- The volatility of the stock price (an estimate of the future volatility), σ
- The risk free interest rate, r
- The dividends that are expected to be paid
-

The current stock price and the strike price determine the value of the call option because the value of the option is based on the difference between the current stock price and the strike price of the option. The higher the stock price exceeds the strike price the higher the value of the option will be. The volatility and the time till expiration of the option are of influence on the option price because a high rate of volatility in combination with a long expiration time gives a higher chance that the price of the stock will exceed the strike price. If there is a sharp decline in the price, the option price cannot be less than zero. Therefore, the holder of the option will only benefit from an increase in the stock price, while his downside risk is limited. Because a higher interest rate will lead to a lower discounted value of the strike price, the risk free interest rate is therefore a factor that will influence the price of the option. If the interest rate is high, the value of the call option will therefore also be higher. The dividend policy of a company also influences the value of the option. This is the case because, a high dividend will lead to a lower rate of growth in the value of the stock and therefore a lower value of the call option (Hefti, 2006).

The table below shows the effect on the price of a stock option when one variable increases. While increasing one variable, all the other variables stay fixed.

Variable	European call	European put	American call	American put
Current stock price	+	-	+	-
Strike price	-	+	-	+
Time to expiration	?	?	+	+
Volatility	+	+	+	+
Risk free rate	+	-	+	-
Dividends	-	+	-	+

Table 3.1 (Hull, 2010)

The value of an option can be 'in the money', 'at the money' or 'out of the money'. If *S* is the stock price and *K* is the strike price, an option is in the money when $S > K$, at the money when $S = K$, and out of the money when $S < K$ (Hull, 2010).

3.3 Option valuation methods

In the world of finance, several methods are available to appraise options. Because the value of an option depends on a number of different variables in addition to the value of the underlying asset, options are difficult and complex to value. There are many different pricing models that are used to value options. In essence, all used methods incorporate the concepts of rational pricing, moneyless, option time value and put-call parity.

Financial option valuation is based on several important principles (Brach, 2003). The first important principle is that there must be assumed that there are no possibilities of arbitrage. No arbitrage possibilities mean that an investor does not have the possibility to create a positive cash flow without paying an extra risk premium for this. In case an arbitrage possibility occurs this possibility is immediately used. Because the arbitrage possibility is over asked an instant correction of the price occurs. The price is now in line with the risk. Only in the situation when arbitrage possibilities do not occur, the price of an option is equal to the costs of the alternative portfolio.
This brings us to the second important principle, namely the assumption that a company is capable of composing a perfect hedged alternative portfolio on the financial markets. This hedge can be created by buying a Δ number of shares in combination with a loan against the risk free interest rate (Hefti, 2006). The combination of the shares and the loan has the same pay off as the option. This results in that the price of the option will be equal to the costs to create this hedge (Trigeorgis, 1999).

Another important assumption is risk neutral valuation. Assuming a risk neutral world gives the right option price for the world we live in, not just for a risk neutral world (Hull, 2010). According to Hull (2010), a person's risk preferences should not affect how options are priced. When options are priced in terms of the price of the underlying stock, risk preferences are unimportant. He states that as investors become more risk averse, stock prices decline. The formula relating option prices to stock prices remains the same. Valuation in a risk neutral world has two features that are present to simplify the pricing of derivatives: the expected return on a stock (or another investment) is the risk free interest rate and the discount rate used for the expected payoff on an option (or another derivate) is the risk free interest rate.

In finance there are several methods that are being used to value options. These are the Black-Scholes model, the binomial options pricing model, the Monte Carlo option model, the Finite difference methods for option pricing and a few more. The first two, the Black-Scholes model and the binomial options pricing model, are the most common and well known, and will therefore be discussed in the following paragraphs.

3.3.1 Binomial model

The binomial model is a very popular technique for pricing an option. This model involves constructing a binomial tree. This is a diagram that represents several different possible paths that the stock price can follow during the life of an option. This model can be done with multiple steps. Because the one-step model as well as the models with multiple steps has the same rationale, only the one-step model will be explained in this paragraph.

A one-step binomial model and a no-arbitrage argument can be explained as follows. Consider a stock price that is currently $20 and it will be $22 or $18 at the end of three months. The valuation concerns a European call option to buy the stock for $21 in three months. The option will have one of two values at the end of the period of three months. The value of the option will be $1 if the stock price will be $22. The value of the option will be zero if the price of the stock turns out to be $18. To value the option, one relatively simple argument can be used to accomplish that. This assumption is that arbitrage opportunities do not exist.

The next thing is to set up the portfolio and the option in such way that there is no uncertainty present about the value of the portfolio at the end of the three months (Hull, 2010). Then it is possible to argue that because the portfolio has no risk, the return it earns must be equal to the risk free interest rate. Because the alternative portfolio has the same characteristics as the option, the value of the option is equal to the costs to create the portfolio (Van 't Hof, 2010). There are only two securities (the stock and the option) and only two possible outcomes. Because of this it is possible to set up the portfolio without risk (Hull, 2010).

If a portfolio consists of a long position in Δ shares of the stock and a short position in one call option. Then calculate the value of Δ that makes the portfolio riskless. The value of the shares is $\Delta 22$ and the value of the option is 1 if the stock price increases from $20 to $22. The total value of the portfolio is therefore 22Δ - 1. When the price of the stock decreases from \$20 to \$18, the value of the shares becomes 18Δ and the value of the option zero. The value of the whole portfolio is then 18Δ. The portfolio is riskless if the value of Δ is chosen so that the final value of the portfolio is the same for both alternatives (Hull, 2010). This means.

$$22\Delta - 1 = 18\Delta$$

or

$$\Delta = 0.25$$

Therefore a riskless portfolio is:

Long: 0.25 shares
Short: 1 option

So if the stock price increases to \$22, the value of the portfolio is:

$$22 \times 0.25 - 1 = 4.5.$$

If the stock price decreases to \$18 the value of the portfolio is:

$$18 \times 0.25 = 4.5.$$

So regardless of whether the price of the stock is increasing or decreasing the value of the portfolio is always 4.5 at the end of the life of the option. This equation shows that Δ is the number of shares that is needed to hedge a short position in one option.

Because there are no arbitrage opportunities, riskless portfolios must earn the risk free interest rate of interest. Let's say that the risk free interest rate is 12% per year. Therefore the value of the portfolio today must be the present value of 4.5, or:

$$4.5e^{-0.12 \times 3/12} = 4.367$$

As assumed earlier the stock price today is \$20. The option price is denoted by *f*. The current value of the portfolio is therefore:

$$20 \times 0.25 - f = 5 - f$$

Following that:

$$5 - f = 4.367$$

or

$$f = 0.633$$

Taken this into consideration it can be concluded that the value of the option must be 0.633 in the absence of arbitrage opportunities. When the value of the option is more then 0.633, the portfolio would cost less than 4.367 to set up and therefore would earn more than the risk free interest rate. If the value of the option would be less than 0.633, shorting the portfolio would be a way of borrowing money at less than the risk free interest rate (Hull, 2010).

To generalize we are considering a stock whose price is s_o and an option on the stock whose current price is f. The life of the option is T. During this life of the option the stock price can increase to S_0u or down to S_0d ($u > 1$; $d < 1$). The proportional increase and decrease in the stock price are: $u - 1$ and $1 - d$. If the stock price is increasing the payoff from the option supposes to be f_u. When the stock price is decreasing the payoff from the option is f_d. As we assumed before we have a portfolio consisting of a long position in Δ shares and a short position in one option. To know how many shares there are needed to make the portfolio riskless, the Δ must be calculated. If the stock price increases, the value of the portfolio at the end of the life of the option is

$$S_0u\,\Delta - f_u$$

If there is a decrease of the stock price, the value is

$$S_0d\,\Delta - f_d$$

These two are equal when

$$S_0u\,\Delta - f_u = S_0d\,\Delta - f_d$$

or

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d}$$

The portfolio is riskless in this case and for the situation that there are no arbitrage opportunities it must earn the risk free interest rate. The equation here above tells us that Δ is the ratio of the change in the option price to the change of the stock price. If the risk free interest rate is r, the value of the portfolio today is

$$(S_0u\,\Delta - f_u)e^{-rT}$$

The costs to set up the portfolio are

$$S_0u\,\Delta - f$$

Following

$$S_0\,\Delta - f = (S_0u\,\Delta - f_u)e^{-rT}$$

or

$$f = S_0\,\Delta\,(1 - ue^{-rT}) + f_ue^{-rT}$$

When substituting from the equation for Δ, the following is

$$f = \mathrm{S}_0\left(\frac{f_u - f_d}{S_0u - S_0d}\right)(1 - ue^{-rT}) + f_ue^{-rT}$$

or

$$f = \frac{f_u(1-de^{-rT}) + f_d\,(ue^{-rT}-1)}{u-d}$$

or

$$f = e^{-rT}\,[p\,f_u + (1-p)\,f_d]$$

where

$$p = \frac{e^{-rT} - d}{u - d}$$

The last two equations enable to give an option a value when stock price movements are given by a one-step binomial model. That there are no arbitrage opportunities in the market is the only assumption that is needed. Taken the numbers mentioned earlier in this paragraph, u = 1.1, d = 0.9, r = 0.12, T = 0.25, f_u = 1 and f_d = 0 this will follow:

$$p = \frac{e^{0.12x3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

and

$$f = e^{-0.12x0.25} [0.6523 \text{ x } 1 + 0.3477 \text{ x } 0) = 0.633$$

This result is the same stated before in this paragraph. This last equation does not involve any of the probabilities of the stock price moving up or down. If the probability of an increasing movement is 0.5, the same option price will follow. This seems unnatural. The reason for this is that the option is not being valued in absolute terms. The value is calculated in terms of the price of the underlying asset. The increasing and decreasing probabilities are already incorporated into the price of the stock. It is not necessary to take them into account again when valuing the option in terms of the stock price (Hull, 2010).

The other important principle in the binomial model, as well for other methods to value options, is the pricing of derivatives in a risk neutral world (Hull, 2010). This assumption states that investors are risk neutral when pricing a derivative. Meaning, risk neutral investors do not increase the required return from an investment to compensate for increasing risk. The world we live in is not a risk neutral world. In this world an investor requires a higher return for a higher risk. But if assuming this is a risk neutral world it would be possible to give the right price to an option in the world we live in, not just for a risk neutral world. This is because when pricing an option in terms of the price of the underlying stock, the risk preferences are not important. When investors become more risk averse, the stock prices will decrease. However, the formula relating option prices to stock prices remains the same (Hull, 2010). There are two features that will simplify the pricing of options. First, the expected return on a stock (or another asset) is the risk free interest rate and secondly, the discount rate used for the payoff on an option (or another asset) is also the risk free interest rate.

When illustrating the result of risk neutral valuation we must return to the following equation.

$$f = e^{-rT} [p f_u + (1 - p) f_d]$$

The last two equations have shown that these give the right price for an option in this situation. Because it is natural to interpret the variable p in the equation above as the probability of an up movement in the stock price, the variable $1 - p$ is then the probability of a down movement, therefore the expression

$$p f_u + (1 - p) f_d$$

is the expected payoff from the option. When interpreting p like this the equation

$$f = e^{-rT} [p f_u + (1 - p) f_d]$$

states that the expected future payoff discounted at the risk free interest rate is the value of the option today. When investigating the expected return of the stock when the probability of an up movement is assumed to be p, the expected stock price at time T, $E(S_T)$ is given by

$$E(S_T) = pS_0u + (1 - p)S_0d$$

or

$$E(S_T) = pS_0(u - d) + S_0d$$

If we substitute this from the following equation for p

$$p = \frac{e^{-rT} - d}{u - d}$$

we obtain

$$E(S_T) = S_0e^{rT}$$

This shows that the average growth of the stock price is the risk free interest rate. By setting the probability of the up movement equal to p is therefore the same as assuming that the return on the stock equals the risk free interest rate. The probability of p is not the same as the probability of an up movement in the real world. The equation above shows that this is the probability of an up movement in a risk neutral world, a world where the expected return on all assets is the risk free interest rate r. Assuming if a world is that way, investors require no compensation for risk and the discount rate to use for the expected payoff is the risk free interest rate. This assumption leads to the valuation for the option to equation

$$f = e^{-rT} [p f_u + (1 - p) f_d]$$

Hull (2010) argues that risk neutral valuation is a very important general result in the pricing of derivatives. According to him it states that when we assume the world is risk neutral we get the right price for a derivative in all worlds, not just a risk neutral world. The examples above have shown that risk neutral valuation is correct when a simple binomial model is assumed for the evolution of the stock price. The fact that risk neutral valuation is correct can be shown regardless of the assumptions that have been made about the stock price evolution (Hull, 2010).

If we take the same example that was first mentioned in this paragraph. The stock price is currently $20 and will move up to $22 or down to $18 at the end of three months. The option is a European call option with strike price of $21 and an expiration date in three months. The risk free interest rate is 12% per year.
The probability of an upward movement in the stock price in a risk neutral world is defined as *p*, this can be calculated from the following equation

$$p = \frac{e^{-rT} - d}{u - d}$$

The expected return on a stock in a risk neutral world must be the risk free interest rate of 12% per year. Assuming this, *p* must satisfy

$$22p + 18(1 - p) = 20e^{0.12x3/12}$$

or

$$4p = 20e^{0.12x3/12} - 18$$

Therefore *p* must be 0.6523. Meaning, at the end of the three months the call option has a 0.6523 probability of being worth 1 and a 0.3477 probability of being worth zero. The expected value of the call options is

$$0.6523 \text{ x } 1 + 0.3477 \text{ x } 0 = 0.6523$$

Because we assume we live in a risk neutral world this value must be discounted at the risk free interest rate. The value of this option is therefore

$$0.6523e^{-0.12x3/12} = 0.633$$

So the value is $0.633. This is the same value calculated before, demonstrating that no-arbitrage arguments and risk-neutral valuation give the same answer. However, it should be emphasized that *p* is the probability of an up movement in a risk neutral world. In general this is not the same as the probability of an up movement in the world we live in. It is not easy to know and apply the correct discount rate to the expected payoff in the real world. If the market requires 16% return on the stock, this is the discount rate used for the expected cash flows from an investment in the stock. But a position in a call option is riskier than a position in a stock. Therefore, the discount rate to be applied for the payoff from a call option is greater than 16%, but it is not known how much greater than 16% it should be. Therefore, using risk neutral valuation is a good method to find out the right price. This is the case, because the expected return on all assets, and therefore the discount rate, is the risk free interest rate (Hull, 2010).

3.3.2 Black – Scholes model

The Black – Scholes model is based upon several assumptions. These assumptions are (Black and Scholes, 1973) (Hull, 2010):

- Stock price behavior corresponds to the lognormal model with μ (expected return on the stock) and σ (volatility of the stock price) constant.
- There are no transaction costs or taxes. All securities are perfectly divisible.
- There are no dividends on the stock during the life of the option.
- There are no riskless arbitrage opportunities.
- Security trading is continuous.
- Investors can borrow or lend at the same risk free interest rate of interest.
- The short term risk free interest rate, *r*, is constant.
- The option is 'European', meaning it can only be exercised at maturity.
-

Some of these assumptions can be relaxed by other researchers. Variations on the Black – Scholes model can be used when *r* and σ are functions of time. Also the formula can be adjusted to take dividends on stock into account (Hull, 2010).

The no arbitrage argument for the Black – Scholes model are analogous to the same arguments mentioned before when discussing the binomial model. Also in this case, a riskless portfolio consisting of positions in an option and one in the underlying stock, is set up. Therefore, in the absence of the opportunity of arbitrage, the return from the portfolio must be the risk free interest rate. Because the stock price and the option are both affected by the same underlying source of uncertainty and stock price movements, a riskless portfolio can be set up. The price of a call option is perfectly positively correlated with the price of the underlying stock and the price of a put option is perfectly negatively correlated with the price of the underlying stock. Because of this, in both cases, the gain or loss from a stock position will always compensate the gain or loss from an option position. With this in mind the overall value of the portfolio at the end of the (short) period of time is known with certainty (Hull, 2010).

Suppose that at some point the relationship between a small change in the stock price and the resultant small change in the price of a European call option is

$$\Delta c = 0.4\Delta S$$

This means that the slope of the line that represents the relationship between Δs and ΔS is 0.6. A riskless portfolio would consist of: a long position in 40 shares and a short position in 100 call options. If the stock price increases by 10 cents, the price of the option will increase by 6 cents and the gain on the shares (60 x 0.10 = \$6) is equal to the loss on the short option position (100 x 0.04 = \$6). Therefore the total wealth is unaltered. This is exactly what a hedged position is intended to do.

There is one important difference between the binomial model and the Black – Scholes model concerning the analysis above. In the Black – Scholes analysis the position that is set up is only riskless for a very short period of time. In theory, it only remains riskless for an instantaneously short period of time. It must be rebalanced or adjusted frequently to remain riskless. So if the relationship between Δs and ΔS is changed to 0.7, an extra 0.1 shares must be purchased for each call option that is sold to maintain a riskless portfolio (Bodie et al, 2010).

The principle of risk neutral valuation that was mentioned in the previous paragraph also applies for the valuation of options by means of the Black – Scholes model. According to Hull (2010), it must be said that this kind of valuation does not state that investors are risk neutral. Instead it states that it is possible to value options on the assumption that investors are risk neutral. Investors' risk preferences have no influence on the value of a stock option when it is expressed as a function of the price of the underlying stock. It explains why the equations do not involve the stock's expected return, μ. Therefore $r = \mu$.

The formula of the Black – Scholes model for a European call option is

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

where

$$d_1 = \frac{\text{Ln}\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

In the equation, c is the price of the call option, S_0 is the stock price, K is the strike price, r is the risk free interest rate, T is the time to expiration, and σ is the volatility of the stock price. Ln is the natural logarithm function.

As you can see $N(d_2)$ and $N(d_1)$ are in the formula that determines the value of the call options. This is the cumulative distribution function of $d_1$and d_2 (Hull, 2010). However, in the original paper of Black and Scholes (1973) $N(d_2)$ and $N(d_1)$ are not explained. Neither did Merton (1973, 1990), Cox and Ross (1976) or Rubinstein (1976). Hull's (2010) textbook also does not explain $N(d_2)$ and $N(d_1)$. Because $N(d_2)$ and $N(d_1)$ are very important to understand, especially in this research, they will be explained.

A paper from Nielsen (1992) did explain $N(d_2)$ and $N(d_1)$ as follows. In the formula of the Black – Scholes model for a European call option the risk adjusted probability that the option will be exercised is $N(d_2)$. $N(d_1)$is somewhat more complicated to understand. The expected value, computed using risk adjusted probabilities, of receiving the stock at expiration of the option, contingent upon the option finishing in the money, is $N(d_1)$ multiplied by the current stock price and the riskless compounding factor. In other words, $N(d_1)$ is the factor by which the present value of contingent receipt of the stock exceeds the current stock price (Nielsen, 1992). Kaeppel (2002), states in his book that Delta is the probability that the option will expire in the money. So, when delta has value of 0.15, the option has a possibility of 15% that it will expire in the money. Lee et al (2010) are stating in their book that Delta, when interpreting and relating to the Black –

Scholes model, can be compared with $N(d_1)$ So, $N(d_1)$ is the probability that the option will expire in the money.

3.4 Conclusion

Stock option valuation is based upon two very important principles. The assumption that there are no arbitrage possibilities and the assumption that every company is capable to hedge itself from risks, a risk neutral world. When assuming these two principles, the true value of the option is determined. These assumptions apply to both methods that are discussed in this chapter, the binomial model and the Black – Scholes model.

The models discussed in this chapter are all based on the valuation of financial options. The fundamentals of the option theory are very important to understand. The theory that is the basis of the method that will determine the possibility that a tenant will renew his contract and assign a value to this possibility is been based upon these fundamentals.

To determine exercise possibilities and value options concerning real estate the models and accompanying parameters must be altered and translated, when using the model for purposes in the real estate market. This will be discussed in the next chapter.

4 Real Options

Next to financial options there are also real options. This term refers to the study of options whose underlying assets (what is obtained or what is given up on the exercise of the option) are real assets (physical capital) as opposed to purely financial options. Also circumstances where real options are present have two characteristics about uncertainties. The first is the future, this is the circumstance where one has no control over. The other circumstance is the ability of the management to respond actively on basis of continuing new insights. When uncertainties are only dependent on future developments than one can speak about a bet and not of an option (Copeland and Keenan, 1998). This chapter gives an explanation of what real options are and will discuss the types and use of real options, the differences between financial options and real options and the use of real options in real estate.

4.1 What are real options?

A building or a factory is an example of a real asset, whereas shares of common stock or a release from a mortgage debt obligation are assets that are purely financial (Geltner et al, 2007). According to Vlek et al (2009), real options are defined as the right to take investment decisions. They relate to the right of the management of a company or a project to, through actions and decisions, react flexible to the circumstances that occur at that time. For a holder of an option, a real option creates flexibility. This flexibility can be used for continuing new insights or changes that can occur in the market that may influence the future cash flows. In contrast to financial options, real options are not traded on the financial markets. Real options can be described as opportunities that the management in the future holds on to. With financial options this opportunity is the right to sell or buy a common stock for a predetermined price. With real options for example, the opportunity is the possibility to delay an investment or the possibility to launch a new product (Nederhorst, 2009).

When taking a look at the traditional valuation models discussed before, like the discounted cash flow method, it can be stated that they are rather static. The consequence of this static characteristic is that uncertainties and/or possibilities have a normal distribution (Trigeorgis, 1999).

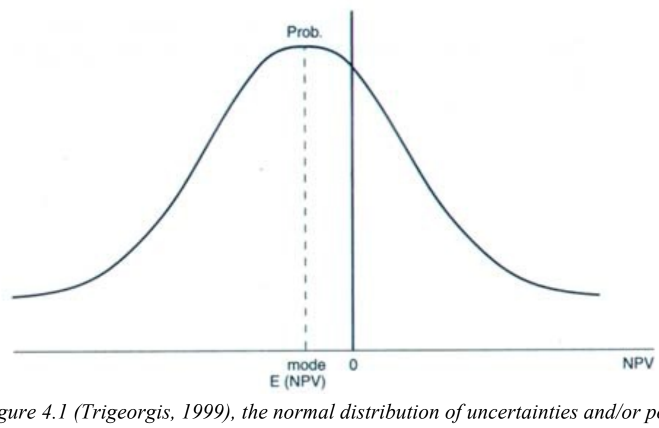

Figure 4.1 (Trigeorgis, 1999), the normal distribution of uncertainties and/or possibilities.

Kranenburg (2000) states that the traditional valuation methods are appraising the investments as if the companies that make these investments are not capable of changing the course of a project, as a response to continuing new insights.

The real option theory assumes that a company is capable of changing the normal distribution to an asymmetrical distribution by hedging the downside and exploiting the possible upside in the future. In the figure below it is shown that through the asymmetrical distribution the normal distribution has a higher average. The difference between the average of the normal distribution and the asymmetric distribution can be seen as an option premium that has to/can be taken into account when there must be a decision whether to invest or not (Kranenburg, 2000). So the asymmetric average is the expected discounted value, according to the static discounted cash flow method, and the option premium. The real option theory improves the decision to invest by taking the flexibility into account. According to Trigeorgis (1999), the option premium should be taken into account with the decision whether to invest or not. A real option gives the holder of the option a certain reactive flexibility. In fact, it offers the possibility of choice to invest, waiting to invest or to disinvest in a project as an answer to continuing new insights (Kranenbrug, 2000).

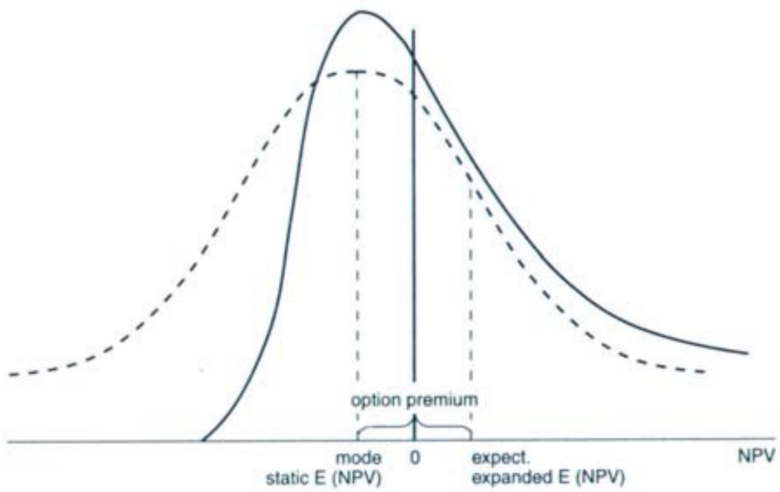

Figure 4.2 (Trigeorgis, 1999), the distribution of uncertainties and/or possibilities when adding flexibility to the project.

4.2 Types of real options

Financial options are very recognizable as they are used and traded daily on the financial markets. Real options are harder to recognize. On the real estate market several real options can be distinguished. However in practice these are mostly overlooked (Hefti, 2006).

According to Kranenburg (2000) there are four main types of real options that can be distinguished. These are:

- The first one is the 'timing option'. This is an option to delay a project. In this case it is about flexibility in timing. For instance, a company has the right, not the obligation, to develop a shopping mall on a piece of land. The company has the possibility to wait with the development when the relation between the market rent and the construction costs are more favorable. This flexibility in timing can be very valuable, especially when there are great uncertainties concerning the costs and revenues that are present with this project. A certain maturity is not necessary. But at a certain moment there is a possibility that the costs that come along with the delay do not weigh up against the value of the option. Besides this, a good look must be taken to what other developers are doing in the market at that time. When the delay is taking too much time, the possibility of the competition taking over the market can arise. When this happens, the value of the option is no longer the only criteria to start with the development. This option of delay is in particular valuable for companies where there is a presence of high uncertainties and a long investment horizon.

- The second real option is the 'growth option'. Uncertainties can be removed by getting extra information concerning the project, new continuing insights will arise. An example is; a company has the possibility to invest a relative small amount in an opportunity to gain more insight in a certain matter that is of great influence on the eventual result of the project. This option can be added up to the net present value, because the option is responsible for the decrease of the downward risk that will occur. This is made possible by the use of new information to maximize the result, through bringing forward, delaying or refraining investments. The growth option can be seen as a call option on a stock that pays dividend.

- The 'flexibility option' is the third real option. This is the option to make an alteration. It concerns the possibility of choice regarding a certain project. For instance a real estate developer has a project that he must realize. When continuing new insights show that the logistic real estate market seems to be a better investment than the office real estate market, it is possible with this option to make a switch to logistic real estate. By changing the plan it is possible to maximize the result of the project. The bigger the freedom is regarding the possibility to change the original program, the higher the value of the option is. The same is valid for the time to exercise the option. The later in the process it is possible to exercise the option, the higher the value will be.

- The fourth real option Kranenburg (2000) mentions is the 'closing option'. This option is the possibility to sell the project for a certain value. This value is the residual value of the project. Putting a limit on the bottom value, and therefore limiting the potential loss, will help to improve the cash flows of the project. The possibility to cancel and sell the project will generate more value. The value of the option can be added to the net present value because it will limit the downward risk. This option can be seen as an American put option.

Vlek et al (2009) also describe in their book the above mentioned forms of real options. In addition to these, he mentions a number of other forms of real options, namely:

- Compound options, these are options on other options. The most important form of this option is the option to divide a project in separate phases. At the end of each phase, the option holder can decide whether to stop with the project or to continue.

- Rainbow- or spread options. These are options where the value is determined through two risky underlying values. An example of such an option is one where the price as well as the production costs are uncertain.

Many real options originate by itself. Some of them can be planned or built in against a certain price (Trigeorgis, 1996). According to Nederhorst (2009) is it possible for a company to create extra value with every project when they make this way of thinking their own. This means creating, planning and using different kinds of options in projects. Triantis and Borison (2001) state that in an economic environment that is characterized by quick alterations, great uncertainties and a growing need for flexibility, it is becoming increasingly important for managers to use methods and tools that give the possibility to react in the right and good manner when continuing new insights concerning investment decisions are arising. Because real estate is a business where long processes with uncertainty are prevalent, the real option theory can be very useful in real estate. The problem however is that the simplified technique of options that is used in the financial world often does not fit the needs for the real estate market. This is because the world of real estate is a very complex one.

4.3 Real options vs. financial options

Real options are similar to financial options in the way that they both give the right to buy or sell an asset at a certain price. But there are important differences between the both options. The parameters on which the value depends are the same with both options. Brach (2003) has compared the different parameters that determine the value of the option. The table below shows the different descriptions of these parameters for real options and financial options.

Financial option	**Variable**	**Real option**
Spot price	S	Discounted future cash flow of the asset
Strike price	K	Costs to buy the asset (investment)
Time to expiration	t	Option term
Volatility	σ	Risk of the asset (difference worst case, best case)
Risk free rate	r_f	Risk free interest rate (time value of money)

Table 4.1 (Brach, 2003), parameters financial options vs. real options.

- For financial options S means the spot price (price of the share). With real options this can be compared to the discounted value of the total future cash flows of the project. This can be calculated with the traditional discounted cash flow method. This is the value of the asset at time 0.

- K is the strike price with financial options and is predetermined most of the time. With real options K, can be seen as the investment that has to be made to exercise the option. In contrast to financial options, the amount of the investment is often insecure with real options. So, in practice, it is the investment that has to be made to get the option.

- The time of the option, t, is for both financial options and real options the time in which it must be exercised. From that point of view, real options seem more like American options because the point of time when the option must or can be exercised is not predetermined.

- Volatility, σ, is seen as the future degree of uncertainty. Both for financial options and real options, a higher volatility will lead to a higher premium for the option. The possibility that the price of a stock will increase is greater, because the down side is covered up to the maximum loss of the premium.

- The, r, is the risk free interest rate, and for both options this has the same meaning and content. An increase of the risk free interest rate will have a positive effect on a financial call option and a negative effect on a financial put option. For a real option is it important to understand what kind of effect the change of the risk free interest rate has on its operational activities (Engels, 2002).

Besides similarities there are also several important differences between financial options and real options. The following differences are derived from several researches and sources of literature (Nederhorst, 2009, Vlek et al, 2009, Van Gool et al, 2007 and Geltner and Miller, 2007).

- One of the most important and biggest differences between financial and real options is the degree of influence that can be made on the parameters that determine the value of the option. With real options these parameters can be influenced, with financial options this is not possible. Important to recognize is that in this case it is also possible to influence the value. This can be achieved by active project management by the option holder. However, one should and must be aware of in which situation influence is possible. Also one must know which parameters make it possible to influence the value in such a way that the value will increase.

- A big difference between financial options and real options is the character of the underlying asset. In contrast to financial options, the underlying asset with real options is physical. With financial options it is more about the difference between the potential and the strike price. As with real options, the environment is more complex than with financial options. In general, management competences, market developments and technological developments will have a greater influence on the value.

- Because financial options are traded on a daily basis, the information that is available to value options and to make decisions to exercise them is at any time available. According to Copeland and Tufano (2004) this is not the case with real options. As they are not traded on a daily basis.

- The concept of time is another difference. The timeline with financial options is clear, there is a certain expiration date of the option. As with real options, this timeline is not clear as such. Besides this, the moment of the exercise will result in another option or exclude other options, so options have and will influence each other reciprocally. Furthermore, the time when the option is exercised and when the financial effects of the exercise are executed are not always at the same time. Prior to the realization of a project, several months or even years have passed at which time continuing new insights could have emerged that could have changed the market.

- The liquidity of real options also differs from financial options. Financial options are easy to trade on the financial markets. On the other hand, real options are in general unique in their kind and therefore more difficult to trade.

- The consequences of when the option is exercised differ from each other. When a real option is exercised, a change in the supply and demand of the market is occurring. When a financial option is exercised the market does not change in terms of supply and demand. This is because when exercising a financial option there will be no larger number of shares on the market. Another difference in the consequences of exercising the option is that a financial option a zero sum game is. What the option holder loses, the writer of the option wins and vice versa (excluding transaction costs). Also when a financial option is exercised there will be no added value. With a real option decisions to invest are the case, so when a real option is exercised there is a possibility of an investment. This will create added value (Witvoet et al, 2007).

- A financial option is always a contract between two separate parties, the option holder and the option writer, while a real option can be seen as an internal mean of control for a project or a company. However, it may be the case that by obtaining the right of flexibility it is necessary to negotiate with other parties.

- With financial options the price of the underlying asset is always clear and available, because shares are always traded every day. For real options the value of the underlying asset is not always clear. Especially, with real estate, because this is not traded and appraised on a daily basis. Besides this, the effects of smoothing and lagging must be kept in mind with appraising real estate.

The differences that have been discussed show that there are significant differences between real options and financial options and that they should be handled differently.

4.4 Real options and real estate

In the real estate market, real options are not uncommon. According to Nederhorst (2009)there are:

- Real options concerning the finance of real estate. When acquiring a real estate finance, several agreements are made, i.e. the time of the loan and the applied interest rate. These agreements can be put in a contract. An agreement to repay a mortgage without penalty can represent a considerable value when interest rates are declining through time. It may be profitable to agree with a higher interest spread, in exchange for the possibility to repay a mortgage without penalty in the future. The higher interest spread can be seen as an option premium.

- The alternative adaptability. Nowadays, the rapid succession of developments is being taken more into account. Every building is being built for a certain function. Through time it can happen that the original function of a building will expire. If that is the case it is of great importance that the building is suitable for alternative adaptability. A simple example can be found in horticulture. For every sort of cultivation there is an ideal height for a greenhouse. For a developer it is possible to take into account that the greenhouse is built in such a way that it can be used to cultivate several sorts of plantation. For this possibility the developer must do an extra investment during the realization of the greenhouse. This extra investment can be seen as the option premium that will generate flexibility in the future for the owner or investor. The flexibility gives the owner and/or investor the possibility to change the cultivation. There is a possibility that this will generate added value or can be used to avoid any losses. It is important to know what this premium (an extra investment at time 0) is worth in the future.

- The option to delay a project. If a project will be profitable or will generate a loss depends on the moment that it starts. Every branch has to do with a business cycle. However, it is not always possible to say and easily to recognize in which phase of the cycle one is located. By means of an option to delay it is possible to have the starting moment of the project at the most desirable moment in the cycle. With much cooperation between municipalities and developers these agreements (options) are made and taken into account. It is decided contractually that a developer has the right to develop a certain parcel for a certain time. If the developer decides not to use this right to develop that certain parcel in that certain time because of disappointing market conditions, he must pay the municipality a compensation for the time that the municipality has reserved the parcel for development.

- Within the real estate market there are many contract deals concerning the lease of office space, retail space, etc. A conventional article in such a contract is the term of the lease. In many contracts a renewal term is agreed. A contract lasts for five years, the renewal will last for another five year. It happens that such a renewal term is described as an option term. The value of this option has not been studied thoroughly. The value of this option is unknown with both parties, the party who leases and the owner. A lease contract with a renewal option is valued the same as a contract without a renewal option. So what is the value of this renewal options? As stated before this is a part of the main question of this research and will be answered later in this research.

Real options must be dealt with in another way than financial options. The trading market of financial options cannot be used for real options in the real estate market. This is because the real estate market is very complex, illiquid, and not transparent (Van Gool et al, 2007). The implementation of the option theory that is used in the financial markets, into the real estate market (the adding of value and taking this into account in an investment decision) seems to have some value (Nederhorst, 2009). According to Nederhorst (2009), it is remarkable that in a market that is financially very driven, the real option theory is not recognized enough in the real estate market. This while there are great uncertainties present in the real estate market such as long term processes, the cyclical character, and the large amount of parameters that can be influenced. Because of these characteristics the real option theory could be very useful in the real estate market.

4.5 Conclusion

The real option theory is discussed in this chapter. The differences between financial options and real options are mentioned and the possibilities and situations in the real estate market where the real option theory can be a suitable method.

Also, there can be concluded that there is one big difference between the traditional appraising methods and the real option method. This difference is that the real option method appraises uncertainty. This is in contrast to the traditional methods, which are static and are not capable to appraise possibilities. Also, they are not capable of assigning a value to these possibilities. That aspect of the real option theory makes it a method that is very suitable to determine the renewal possibility and to assign a value to this possibility, as this is an uncertainty in real estate.

As mentioned in chapter 1 the main question of this research is *: 'In what way is it possible to quantify and value the possibility of a lease renewal?'*. In the next chapter the real option theory that is discussed in this chapter will be translated, so that it can be applied to real estate lease contracts. With this translation to lease contracts, it is possible to quantify the possibility that a tenant will renew his contract. Together with this, a value will be assigned to his possibility. The model that is used to determine the exercise possibility and value is discussed as well as the most important characteristics of it.

5 Translation to real estate lease contracts

If the real option theory must be applied to leasing contracts in real estate, the model that is used must be changed in some way. Simply said, the input parameters must be altered for the specific use of the model.

In this chapter, the model that is used to quantify the possibility of a lease renewal and to value this possibility is discussed. First, the explanation of the choice of the model that is used is given. After this, the input parameters are discussed and there will be an explanation of how these parameters are altered and interpreted for the specific use for real estate lease contracts. After discussing all the variables, the most important one, volatility, will be discussed. In the original paper of Black and Scholes (1973) $N(d_1)$ and $N(d_2)$ are not discussed nor explained, in this chapter the interpretation for real estate, especially lease contracts, of $N(d_1)$ and $N(d_2)$ is discussed. Also some case examples will be shown. This will show what the outcome of the renewal possibility is and the value of this possibility when a contract is entered in the model under different circumstances. Finally, this chapter will end with an interpretation of the outcome of the case.

5.1 The model

As explained earlier in this research, in chapter 2, it was concluded that the traditional methods do not suffice to assign a value to the possibility of a lease renewal and the renewal possibility. The gross initial yield method and the net initial yield methods use variables that limit the possibilities. These methods are predominantly used to appraise current real estate objects. They do this by means of a large amount of comparable data and make comparisons. The methods are mainly applicable for objects of projects with a low level of uncertainty. The applicability is very wide and simple in use, because of the limited number of variables. The result is a comparable number, not an exact scientific calculated value. Besides this, the assumptions that are made are a snapshot of the time of the appraisal (Nederhorst, 2009). The discounted cash flow method goes a step further. Multiple variables must be assumed. For example, the expenses in time, the income in time, the interest rate, inflation, the discount rate, the costs of money, etc. Because of the current technology, discounted cash flow methods are relative easy to make. The appraisal is relatively scientific. However, it is also a snapshot of the time of the appraisal. The level of uncertainty that is taken into account in these models is in general the negative aspect of uncertainty (Nederhorst, 2009).

In chapter 3, the option theory was explained and the two models that are the most frequent used ones, the binomial model and the Black – Scholes model, were discussed. As already mentioned in chapter 3, the binomial model calculates the value of the option by means of a binomial tree. Discounting takes place with the risk free interest rate and with help of the volatility of the underlying asset the up- and downside movements (uncertainties) are calculated (Hull, 2010). According to Nederhorst (2009), this method is relatively easy to use. To make a calculation that is as accurate as possible it is necessary that the underlying time steps are as little as possible. The level of accuracy will be considerably higher. However, the complexity will also increase (Frusch, 2008).

Of the methods that are discussed, the Black – Scholes is the most exact and scientific to assign a value to a real option. In particular for a project where there a considerable amount of uncertainty is present, the Black – Scholes model is suitable. Because of the level of exactness, the model is dependent on exact input data. Volatility is a very essential input variable in this model. Data on volatility in the real estate market is still insufficiently available. In addition to this, it is questionable if the required data on volatility is present for each individual project. Every real estate project has its own features and characteristics.

Each method has its own specific characteristics and uses different input variables. When comparing the methods, it becomes clear that each method has its advantages and disadvantages. The simplest method is the comparable method (which is not discussed in this research) and the most complex one is the Black – Scholes method. The Black – Scholes method is also the most suitable one to use when there are great uncertainties present within the project and the value must be calculated with a great level of exactness.

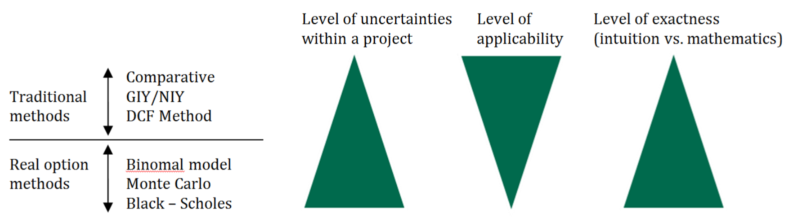

Figure 5.1 (Nederhorst, 2009), applicability of the methods.

According to Nederhorst (2009), some contradiction is present. With projects where a lot of uncertainties are present, one wants to have a great amount of exactness. However, the method which is most suitable for that is also the least usable. The more uncertainties that are present, the more suitable it is to apply the real option theory, but the less easy to apply.

To determine and value the possibility of lease renewal, the Black – Scholes model is chosen to use for this research. One precondition that is mentioned in the original article of Black and Scholes (1973) is that the option must be a European one. From this point of view a lease contract can be seen as a European option. As a lease contract lasts for a certain predetermined time, the contract will end at a certain date. The option to renew can only be exercised at the time the original contract ends. In finance, a European option can only be exercised at the expiration date of the option (Hull, 2010). The Black – Scholes model is based on an endless amount of steps. This is an advantage over the binomial model. The binomial model will become very complex and very large when using a large number of steps. As stated before, the Black – Scholes model is very suitable in situations where a level of great uncertainty is present. This is the case in this research, the possibility of a lease renewal is one where levels of great uncertainty are present. The result must also be one which is exact and as objective as possible. Therefore, the levels of intuition that are present in the model must be as low as possible. As the applicability level is low with the Black – Scholes model, the input parameters must be interpreted correct. Otherwise, the outcomes of the model are useless. Combining the factors described above, the model that is regarded as most suitable for this research is the Black – Scholes model.

5.2 Parameters

In chapter 4, paragraph 4.3 discussed the differences between the parameters of financial options and real options. In real options for real estate. many differences are present. The interpretation for the parameters for a closing option is different than with a lease contract renewal option. But how must we interpret the parameters of the real option theory to real estate lease contracts?

The table below shows how the input variables must be read so that they apply to real options.

Variable	**Real option**
S	Discounted future cash flow of the asset
K	Costs to buy the asset (investment)
t	Option term
σ	Risk of the asset (difference worst case, best case)
r_f	Risk free interest rate (time value of money)

Table 5.1 (Brach, 2003), parameters real options.

The interpretation of the input variables for a real estate lease contract are as follows:

- The S for real options is the discounted future cash flow of the underlying asset. If it concerns a five year lease, the cash flows must be discounted for five years, using the r_f. The underlying asset is the lease contract that will start after five years. When adding up the discounted cash flows, the result is the discounted future cash flow of the asset, S, at year one.

- The second input variable is the K, the costs to buy the asset. This is the strike price. When relating this to lease contracts, K is the market value of the contract against current conditions. So, the discounted value of the lease contract for the next five years. When the option is exercised, these are the costs that the option holder must invest to obtain the asset. This is the value of the contract at the time of exercise, year five.

- The option term, T, is the duration of the rental lease contract of the underlying asset. What number this is, depends on the length of the contract that is closed.

- σ is the risk of the asset. For the option holder in this case, the risk is the possibly change of the annual paid rent. This possibly change is the volatility. To determine σ as an input variable in the model, the standard deviation is calculated of the collected historical data of (market) rental prices per m^2 per year. Because real estate objects are unique in its own kind, one who wants to use the model must calculate the standard deviation of each underlying asset separately. Of the last decade(s), the historical data of (market) rental prices of the different real estate sectors is available.

- The last input variable is the risk free interest rate, r_f. For the risk free interest rate, the ten year government bonds are often used to determine the risk free rate (D'argensio and Laurin, 2008). But according to Damodaran (2008), the risk free interest rate must be derived from government bonds with the same time horizon as the investment. Thus, the risk free interest rate for a five year time investment, in this case a lease contract, has to be the expected return on a five year state bond and for a ten year lease contract the ten year government bonds must be used. So, the risk free interest rate that must be used depends on the lease term.

5.3 Volatility

When taking a look at the variables in the model, the sharp reader will notice one thing. All of the variables, except one, are given and sure numbers. The one variable that is not sure is σ, the volatility. And that is just the reason why this is the most important variable. When entering the volatility of an asset in the model, this number is always based on historical data. As it already happened in the past. But when an option pricing model, like the Black – Scholes model, asks for volatility it really needs to know the future volatility of the asset, not the historic volatility. This is because when an option is being valued and the exercise possibility is determined, something that will happen in the future is being determined. With this, uncertainty is quantified, and uncertainty says something about the future, not the past. Therefore, it is the future volatility of the asset that determines the price of an option. But unfortunately, this is something that will not be known until the expiration. So, in order to truly know the value of an option the future volatility of the underlying asset must be known. Beckers (1981), also states that the volatility is the most difficult variable to estimate. He states that the practical relevance of the Black – Scholes model as an option pricing formula depends on the ability of the investor to forecast the future asset price volatility.

Another reason why the volatility is the most important input variable is that it is the variable which has the biggest influence on the price of the option. The level of volatility indicates how uncertain we are about future asset price movements. When the volatility of an asset increases, the possibility that the asset will perform very well or very poorly increases. For the owner of this asset, these two outcomes tend to offset each other. But, this is not the case for an owner of a call (or put). The owner of a call benefits from price increases but has limited downside risk in the event of price decrease because the most the owner of the call can lose is the price of the option. The opposite is true for the owner of a put. Therefore, the value of both call and put will increase as volatility increases (Hull, 2010).

As stated before, to know the true price and therefore also the true possibility of the exercise of an option, the future volatility must be known. When using this model in practice, rental market values must be forecasted to calculate the future volatility. In this research, we assume the volatility used in the case examples is forecasted.

5.4 $N(d_1)$ and $N(d_2)$

This research focuses on the discussion on the extent to which it is possible to determine the possibility of a lease renewal and to assign a value to this possibility. As stated before, $N(d_1)$ and $N(d_2)$ are the outcomes of the two possibilities that result from the Black – Scholes model. In finance, $N(d_1)$ is the factor by which the present value of contingent receipt of the stock exceeds the current stock price (Nielsen, 1992). Simply said, it is the probability that the option will expire in the money (Nielsen, 1992). When relating this to real estate lease contracts, it can be said that this is the possibility that the rental market value per m^2 per year is higher than the rent the tenant will be paying per m^2 per year when he renews the lease contract. With a financial option, $N(d_2)$ is the risk adjusted probability that the option will be exercised. In the case of real estate lease contracts, $N(d_2)$ can be seen as the possibility that a tenant will renew his contract under the same conditions as the first contract.

5.5 Conclusion

This chapter concludes that the most suitable model to quantify the lease renewability and to assign value to this possibility is the Black – Scholes model. The Black – Scholes model is very suitable to use for projects where there is a great level of uncertainty present. Of the available models the Black – Scholes model is (on of) the most exact ones. To use the Black – Scholes model for the real option theory relating to lease contracts, the parameters must have another interpretation. The interpretation of these parameters is as follows: the S is the discounted future cash flow of the investment, K is the cost to buy the asset, T is the duration of the lease contract, σ is the volatility (risk) of the asset and r_f. is the risk free interest rate. The most important parameter is the volatility of the asset, σ. There are two reasons why the volatility is the most important input parameter. The first one is that this is the only parameter that is not a given and sure number. The second reason is that it is the parameter that has the biggest influence on the price of the option.

In the next chapter, a simulation of different scenarios will follow. These scenarios of cases will show what the influence of changes in the different parameters is on the option price and exercise possibility. An interpretation of the outcome of the simulation is also given.

6 Simulation

This chapter gives an analysis of the simulation that is done in this research. What is the influence of changes in parameters on the price of the option and the exercise possibility? This question will be answered by showing several scenarios of cases where one or more parameters are changing at the same time. By doing this, it will become clear what influence these parameters have on the price of the option and the exercise possibility when they are changing. First, the method is explained. Following that, the outcome of the simulation is discussed, followed by an interpretation of this outcome. After this new simulations are discussed as well as the interpretation of these. This chapter will end with a small conclusion.

6.1 Survey

As discussed earlier in this research, to know the true value of an option the future volatility must be known. This is because when an option pricing model, like the Black – Scholes model, asks for the volatility of an asset, it needs the future volatility. When an option is being valued and the exercise possibility is determined, something that will happen in the future is being determined. When mentioning uncertainty, one is always speaking about the future. The past cannot be uncertain, as it already happened.

To determine the future volatility, a survey has been made for this research. This survey can be found in appendix 1. The intention of the survey was to determine the expected future volatility of three different assets. As we cannot know the future volatility, forecasting is the best possible solution. To determine the forecasted future volatility, the survey was sent to over twenty real estate experts such as appraisers, bankers, investors, brokers and consultants.

The volatility of an asset is the standard deviation of the value. In this case, a lease contract, the volatility is the standard deviation of the price per m^2 per year. In the survey three different assets can be found. The expected rental value per m^2 per year for the upcoming three years is asked of those three different assets. For the volatility in the model created in this research a volatility for five years is chosen. As it is difficult to forecast five years ahead, it is asked to forecast three years ahead. To determine the volatility, the standard deviation is calculated from last year, the current year and upcoming three years. The rental value per m^2 of the upcoming three years, are the average values per year of the answers given by the real estate experts that are questioned.

By conducting this survey, the research will get an empirical element in it. So far, the research is solely theoretical. Unfortunately, the response from the survey was very slight. Of the more than twenty surveys that were sent to the real estate experts, only two were filled in and sent back. As a result of this, the research is only pure theoretical and the volatility in the model and different cases and scenarios are assumptions that are made. However, when using assumptions, the way the different input parameters influence the value of the option and the exercise possibility does not change.

6.2 Simulation outcome

To demonstrate how the real option model works, that is based on the Black – Scholes model, three sets of three case examples are given and discussed. The first set of three cases shows us what influence the height of the market rental value is on the value of the option and the exercise possibility. The second set shows us what influence the height of the volatility is on the value of the option and the exercise possibility. The first two sets assume that the moment of exercise is in five years. Therefore, the value and exercise possibility of the option is based upon that assumption. The third set will show the influence on the value and the exercise possibility when the moment of expiration changes.

The first set of the three cases is shown in appendix 2 and in the table below a short summary is given. The value of the option is under the influence of the height of the rental market value. However, the influence is only in absolute terms of numbers. Relatively, nothing changes with the relation between the height of the market rental value and the price of the option. In the first table, the market rental value is $100,000 and the price of the option $38,196. If we take look at the second table, the annual rental market value is changed to $200,000. The value of the call option has increased to $76,392. This shows that the price of the option, together with the rental market value, doubled. In the third table, the market rental value is divided by three, from $100,000 to $33,333. With this, the value of the option becomes $ 12,732. This is exactly one third of $38,196, as this is the value of the option when the market rental value is $100,000. This shows that the value of the option increases or decreases with the same percentage as the market rental value increases or decreases. This means that the height of the market rental value has no influence on the height of the value of the option in relative terms, merely in absolute terms. The above is correct, if we assume that the volatility does not change with the increase or decrease of the rental market value. The cases also show that the height of the market rental value is not of influence on $N(d_2)$, the probability of exercise. This is because the probability of exercise is dependent on the volatility of the asset and not on the price of the asset. Therefore, the probability of exercise remains the same in each case at 45.7%, no matter the changes in the height of the market rental value.

Market rental value	σ	Time till expiration	C	Exercise possibility
€ 100,000	10%	5	$ 38,196	45.7%
€ 200,000	10%	5	$ 76,392	45.7%
€ 33,333	10%	5	$ 12,732	45.7%

Table: 6.1: Market rental value cases

The second set of the three cases shows the influence of the variety in volatility. They can be seen in appendix 3. A short summary of the outcome of the cases is in the table below. In the first table, we see that the volatility is 1%. With this volatility, the value of the option is $4,021 and the exercise possibility 51.3%. If the volatility is increased to 10%, the value of the call options also rapidly increases to $38,196. The possibility that the option is exercised decreases to 45.7%. In the last table, the volatility is increased again to 20%. The value of the call option has again rapidly increased to $75,710 and has almost doubled. The exercise possibility has also, again, decreased somewhat to 41.2.%. So, we see that when volatility increases the value of the option also increases. Together with this increase of the value of the option, we see a decrease in the possibility that the option will be exercised.

Market rental value		σ	Time till expiration	C		Exercise possibility
€	100,000	1%	5	$	4,021	51.3%
€	100,000	10%	5	$	38,196	45.7%
€	100,000	20%	5	$	75,710	41.2%

Table 6.2: Volatility cases

In appendix 4, the third set of the three cases is shown. In the table below is a short summary of the outcome of this set. This set shows what the influence on the value and the exercise possibility of the option is when the time till expiration is changing. In the first table, the time till expiration is five years. In this case, the starting point is the time when the first contract is signed. The value of the option is $38,196 and the exercise possibility is 45.7%. The second table shows what will happen when the time till expiration is shortened to three years, the current contract runs for two years. The value of the option has increased to $48,221 and the exercise possibility has also increased to 52.8%. In the third table, the time till expiration is shortened again, now to one year. The current contract runs for four years. The value of the option increases, again, to $59,863 and the exercise possibility also increases again to 59.7%. We see that when the time that rests till the moment of expiration decreases, the value of the option as well as the possibility that the option will be exercised increases.

Market rental value		σ	Time till expiration	C		Exercise possibility
€	100,000	10%	5	$	38,196	45.7%
€	100,000	10%	3	$	48,721	52.8%
€	100,000	10%	1	$	59,863	59.7%

Table 6.3: Time till expiration cases

In all three case examples of the two sets the following assumptions were made.

- For t, we assumed the length of the contract is five years. Of course, is it possible to vary this length. It is easy to alter the model so that it can be used for one year, three year or ten year lease contracts.

- σ, is the volatility of the underlying asset. This means the volatility of the market rental price per m^2 per year of the underlying asset. We assume the given volatility is the future volatility, because by using the future volatility the value of the option will be the true value.

- The risk free interest rate, r_f, which is used in the model is 2%. In practice, the actual real risk free rate must be used to get the real value and renewal possibility.

Note that the given numbers for the parameters are not the same as in reality. To show what the influence on the value of the option or the exercise possibility is when parameters increase or decrease, it is not important that the numbers that are used reflect the current reality. The mechanics of the model do not change when assumptions are made.

6.3 Interpretation of the cases

If we take the outcomes of the three sets, each with three different cases, we can interpret the following. The height of the market rental value does not influence the value of the option. Also, it has no influence on the exercise possibility of the option. The changes that do occur with the option value, when changing the height of the market rental value, are only in absolute terms as the value changes with the same ratio. The exercise possibility does not change at all, but remains the same.

When the volatility increases , the value of the option increases as well. At the same time, the exercise possibility decreases. This is logical, because volatility is the reflection of risk (risk is not a purely negative thing, there is also a risk present that the option will be in the money). It is a risk that the market rental value will increase or decrease. When the possibility is high that an option will be in the money, it is obvious that the value of that option is high. And with a high volatility, the possibility that an option will be in the money is larger than with a low volatility. Therefore, the value of the option will increase. When there is a high risk that the market rental value will strongly increase, and the tenant has an option to renew his contract under the same circumstances as the first contract, the option will be deep in the money as the rent is lower than the market rental value. This means the tenant may be willing to buy the option for a large sum of money. However, with a high volatility, the risk is also larger that the option will be out of the money. The market rental value will be lower than the rent the tenant is paying in his current contract. It is obvious that the tenant will not renew his contract under the same circumstances. This means the option will not be exercised. Therefore, the exercise possibility will decrease when the volatility increases. So why does the price of the option rise and not fall? The owner of the option (call) benefits from this price increase, but has limited downside risk when the price is decreasing. This is because the only amount of money the owner can lose is the amount that he has paid for the option. The only thing he has to do when the option is out of the money is not to exercise the option. Therefore, the value will increase as volatility increase (Hull, 2010).

6.4 New case simulations

The closer the date comes, the more the value of the option and the exercise possibility increases. When seeing this in the third set of cases it should be interesting to combine the second and third set of cases in a new fourth set of cases.

In this set, we see a case where there is a decreasing volatility in combination with a decreasing time till expiration. This set can be found in appendix 5 and a short summary of the outcome in the table below. There are two things that stand out. The first is the rapid increase in exercise possibility. The second thing is that the decrease of the option value turns into an increase. In the first table, the volatility is 10% , the time till expiration 5 year, the value of the option $38,196 and the exercise possibility 45.7%. The case in the second table has a volatility of 6%, the time till expiration 3 year, the value of the option decreases to $33,173 and the exercise possibility increases to 59.3%. In the third case the volatility is now 2%, the time till expiration is 1 year, the value of the call option has increased again to $35,901 and the exercise possibility has rapidly increased to 96.2%.

Market rental value	σ	Time till expiration	C	Exercise possibility
€ 100,000	10%	5	$ 38,196	45.7%
€ 100,000	6%	3	$ 33,173	59.3%
€ 100,000	2%	1	$ 35,901	96.2%

Table 6.4: Volatility in combination with time till expiration cases

Because the value of the option has suddenly increased again in the third case of the fourth set, a fifth case is done. This case is done to show what the influence of the value of the option and the exercise possibility is when the time till expiration is 1 year. This case can be seen in appendix 5. In the table below a summary of the sets is given. In the first table, the volatility is 10%, the value of the option is $59,863 and the exercise possibility is 69.7%. In the second table, the volatility has decreased to 6%. The value of the option has also decreased, to $45,683 and the exercise possibility has increased to 70.2%. In the third case, the volatility has decreased again, to 2%. And also, the value of the call option has decreased, to $35,901 and the exercise possibility has increased again to 96.2%.

Market rental value	σ	Time till expiration	C	Exercise possibility
€ 100,000	10%	1	$ 59,863	59.7%
€ 100,000	6%	1	$ 45,683	70.2%
€ 100,000	2%	1	$ 35,901	96.2%

Table 6.5: Volatility in combination with expiration in 1 year cases

6.5 Interpretations of the new cases

As already concluded in the first three sets of cases, when volatility increases the value of the option also increases and the exercise possibility decreases. The reverse is true when volatility decreases. Also, when the time till expiration decreases, the value of the option as well as the exercise possibility increases.

In the new two sets of three cases these changing factors are combined together. Here, we see that a decreasing volatility together with a decreasing time till expiration leads to a rapidly increasing exercise possibility and to a somewhat stabilizing value of an option.

So, according to the model, when the volatility in market rental prices is low and the current lease contract is almost running to an end, there is a high possibility that the tenant will renew his contract. Again, this model only bears possible future changes in market rental values in mind to determine the value of the renewal option and the exercise possibility. And taking this into consideration, it sounds logical that when volatility is low and the time till expiration is short that a tenant will renew his contract.

When the market rental value is lower than the price of the current contract at the expiration date, only a fool would renew his contract under the same circumstances. If he would renew the contract, he would lose money. On the other hand, when the market rental value is higher than the price of the current contract at the expiration date, only a fool would not renew his contract under the same circumstances. If he would renew in this case, he would gain money. But, when volatility is low, the possibilities that the market rental value will be higher or lower than the contract price are both slim.

Therefore, the possibility to lose or to gain, in terms of money, when renewing the contract under the same circumstances is also low. On the other hand, the possibility to lose or to gain, in terms of money, when not renewing the contract under the same circumstances are also low at the same time. So, when there is nothing to lose or to gain in short terms by not renewing the contract, i.e. to move, why move?

6.6 Conclusion

If the option models are compared with each other and related to the subject and aim of the research, the Black – Scholes model is the most suitable model to use in this research. Of the input parameters of the Black – Scholes model, volatility is the most important one and has the largest influence on the option value and exercise possibility.

The most important finding of the case examples is that the combination of a low volatility with a short time till expiration has the highest exercise possibility. The combination of volatility and time till expiration has the biggest influence on the height of the exercise possibility. The theory behind this is, when volatility is low the possibility is also low that the market rental value is higher or lower than the current contract rent. Therefore, the possibility that there is something to gain or lose, in terms of money is low by not renewing the contract .From the point of view of this model, financial gain or loss, there is no need not to renew the contract. Therefore, the possibility of exercise is high.

7 Conclusion

In this last chapter, the final conclusion is discussed, in which the two main questions and the sub questions will be answered. This research will end with a discussion on the subject about how to apply this model in practice. Because the discussion will be about the practical relevance and use of the theory and model, the last sub question will be answered here. Finally, some recommendations are given that have been derived from this research.

7.1 Final conclusion

After discussing the traditional methods that are currently being used in the real estate market to appraise, it became clear that these methods were rigid and static. They are not capable of determining uncertainties. Risk is processed in the risk premium, which together with the risk free interest rate makes up the required return. For the traditional methods, the required return is the reflection of risk. And the risk premium is currently based upon and determined by experience, subjective and intuitive ways (Hishamuddin, 2000). When an appraiser or investor assumes the risk of vacancy is high in the future, the tenant will not renew his contract, he just increases the risk premium to a level he finds suitable. The required return increases, reflecting the risk. With this new increased required return a new value of the object will be determined. According to Trigeorgis (1999), these methods are static because they assume that uncertainty has a normal distribution. Therefore, it is not possible with the traditional methods to determine uncertainties, let alone to value uncertainty.

By concluding this, the following sub questions are answered.

- *How is the possibility that a tenant will renew his lease contract now determined?*

- *In which way is this possibility currently taken into account in appraising real estate?*

- *Can the possibility that a tenant will renew his contract be determined by means of the traditional methods?*

So, it is not possible with the traditional methods to determine uncertainties, let alone to value uncertainty. According to Trigeorgis (1999), these methods are static because they assume that uncertainty has a normal distribution.

So, if the traditional methods are not capable of answering the main question of this research, a look at alternative methods must be taken. When someone has the choice to renew his contract he has a choice, an option. It is a right and not an obligation. A financial option has this same characteristic. A holder of an option to buy a stock for a certain price has the right, and not the obligation, to buy that stock at a certain time and for that certain price. These financial options also have a value. Therefore, they can be valued. Several methods are capable of doing this. For this research, the Black – Scholes method is used to quantify, and at the same time value, the possibility that a tenant will renew his contract. To apply the financial option theory for real estate, the underlying assets are real assets (physical capital) as opposed to purely financial options. This form of the financial theory is called the real option theory.

This brings us to the next sub questions of the research. The answer to the following questions,

- *How is it possible to apply the (real) option theory to determine the possibility that a tenant will renew his contract?*

- *Is it possible to translate this theory to real estate lease contracts?*

is: The real option theory can be applied to several purposes in real estate. The main difference between the traditional methods and the real option theory is that the real option theory assumes that a company is capable of changing the normal distribution to an asymmetrical distribution by hedging the downside and exploiting the possible upside in the future. It is capable to determine possibility and assign a value to it. For every purpose the interpretation of the parameters must be altered. This is because the underlying values must be correct in order to get a result that is mathematically correct. By concluding this, the answer to the first main question,

- *In which way is it possible to quantify the possibility of a lease renewal?*

is: It is possible to alter the Black – Scholes model in such a way that it can be applied to the real option theory for purposes in real estate. Therefore, when the parameters are altered in a way that they apply, and are correct, to real estate lease contracts, the model is suitable. In finance, the Black – Scholes model calculates the exercise possibility of the option. So, when using this for real estate lease contracts, the exercise possibility that is calculated is the exercise possibility that the renewal option will be exercised, i.e. the possibility of a lease renewal.

Now that the first is answered, the second main question can be answered. The answer of the second main question,

- *To what extent is it possible to assign a value to this possibility?*

is: When the Black – Scholes model calculates the exercise possibility, at the same time the value of the possibility is calculated. So, by calculating the exercise possibility, the value of this possibility is instantly known. Just as the Black – Scholes is the method, and way, to quantify the exercise possibility of a lease renewal, it is also the method to assign a value to this possibility.

That leaves us with the last sub question in this paragraph that needs to be answered. In chapter 5, the model is explained and several sets of cases were shown. The aim of the cases was to show the influence when specific parameters are changed. The answer to the last question,

- *Which factor(s) determine(s) the exercise possibility and the value of the option?*

is: In chapter 5, the translation to real estate contracts is explained together with the explanation of the model and the different sets of cases. The aim of the cases is to show how the exercise possibility and the value of the option are influenced by changing a specific parameter. The conclusion of the cases is, that the specific combination of a low volatility and a short time till expiration has the highest exercise possibility. When volatility is low, then at the same time there is a low possibility that the market rental value is higher or lower than the current contract rent and vice versa. In this case, of low volatility, there will be little chance that there is something to gain or lose, in financial terms by not renewing the contract. Therefore, the exercise possibility is high. The value of the option is, predominantly, determined by the height of the volatility. Because, when the volatility is high, the possibility that there will be great financial gain is also high. It is obvious that one is willing to pay more when the risk of financial gain is high instead of a low risk of financial gain. So, the factor that has the biggest influence on the height exercise possibility is the combination of volatility and time till exercise.

7.2 Discussion

This research has produced a model and theory about the possibility that a tenant will renew his contract. This is done by means of the real option theory. The model that is used to apply the real option theory is the Black – Scholes model that Fisher Black and Myron Scholes (1973) have created, which won them a Nobel prize. As the real option theory is a rather new method in real estate and it is practically not used in practice, the following question arises,

- *In what way can the real option theory for real estate lease contracts be applied in practice?*

The main question of this research was derived from a problem, encountered by the Valuation Advisory department at CBRE Netherlands. The problem is, that banks, investors and other financial institutions are asking, or demanding, the large international real estate consultancy firms for valuation reports that are more comprehensive, thorough and detailed and must have a better research foundation than before. Shortly stated, clients want a better risk analysis. Therefore, this research is quantifying risk in an objective way, in contrast to the more, intuitive and experienced based ways that are currently used in the real estate market (by the real estate consultancy firms). These traditional methods are also prone to subjective influences. So, the question can and must be asked, is this the right way to give an objective valuation advice?

As the real estate market and their professionals, in consultancy, are used to this (personal) experienced based method, this abstract, rational and mathematically based method can be seen as strange and looked down upon. How must this method interpreted? And how can it be applied in practice?

If one would calculate and determine the renewal possibility, it would be obvious to do this on the basis of historical data. But when doing this, the only thing that is calculated and determined is a percentage of tenants that have renewed their contract in a certain period of time in the past. They have had their own subjective and financial motives to do this, as the motives to renew or not to renew are personal and therefore unique. It is possible to see some relations and similarities between tenants who have renewed their contract. But what does that say about the future and other tenants? As they say, results from the past do not guarantee future results? If a client asks for a possibility that a tenant will renew his contract, the client wants to hear a forecast. Instead of using a method that derives the probability on historical data from a very specific period of time and specific tenants with their own subjective motives to renew, why not use a model that is based on the future and is capable of processing uncertainties?

The point of view of which the model is calculating the exercise possibility is that of financial gain or loss. The major input variable is the volatility of the market rental price of the underlying asset, i.e. the uncertainty of the price per m^e per year of the underlying asset. Geographic location, business sector, floor surface, and all other characteristics present, are not taken into account in this model.

It can be stated that the model assumes that the tenant is a ‘homo economicus’. A ‘homo economicus’ is a person who is rational and narrowly self-interested, who has the ability to make judgments toward subjectively defined ends. When using these rational assessments, the ‘homo economicus’ attempts to maximize utility as a consumer and economic profit as a producer (Rittenberg and Tregarthen, 2009).

According to the model, the tenant is a “homo economicus”, but is he in practice? The main factor in the results of the model that determines high exercise possibility, is the combination of low volatility and a short time till expiration. These factors alone are also an important influence on the exercise possibility. This is not tested in practice. So, if it is true that more tenants will renew when the volatility of their rent is low in combination with a short time till their current contract expires. When the results of the model are in accordance with the results of the follow-up research, the model is correct and can be used in practice.

The model and the results of the cases are rational and abstract. It is difficult to interpret the exercise possibility and the value of the option. Therefore, it is difficult to use directly in practice. In particular for the large international real estate consultancy firms this model is difficult to apply. However, on the other side, the, for the large investors, there are possibilities to apply this method in practice. The large investors, institutional and private, are looking with a more quantitative perspective towards the real estate market, this in contrast to the more subjective and intuitive perspective that is found with the real estate brokers.

The institutional investors have financial responsibilities towards their financial liabilities, such as pensions or insurances. The private investors have financial responsibilities towards their shareholders, they want to see profit and/or dividend. Because of these responsibilities and liabilities, investors have a different look towards real estate. A more quantitative one.

The real estate brokers/consultants litterally are standing with their feet in the market and talk to the local broker, the small local investor i.e. trader and the local appraiser. They know the local real estate market as their back pocket. So, their opinion and knowledge is based upon years of experience. One might expect that this model, which is abstract and rational, will be of no kind of use for a relatively small local investor or for a broker. Also, this model may not be applicable in such a small local market, as that local market is so specific. Therefore, it is completely logical that the parties that are operating in a local or smaller market trust upon their years of experience and intuition.

Even for large consultants, such as CBRE, this model is difficult to apply. As the consultancy branch of real estate does not own or manage real estate portfolios their perspective is different. A reason for this could be that they have no financial liabilities that are depending on financial real estate performances. There can be reason to believe that when a company owns real estate and responsibilities and liabilities depend on the financial performance of that real estate, that company does not satisfy with advice that is based upon experience i.e. subjective knowledge. They want assurances based upon rational and objective knowledge.

Therefore, this model is made on behalf of a problem stated by the Valuation Advisory department of CBRE Netherlands, but it seems that this model is not of the use they hoped for. This model is better applicable for a company that has a more quantitative and rational perspective on real estate. These companies are large institutional and private investors, companies, where responsibilities and liabilities are dependent on the financial performance of real estate. In these companies there is little room for decisions that are made on behalf of intuition and subjective knowledge. The responsibilities are too important. Therefore, the decisions must be made upon knowledge that is based upon rationality and objectivity. As no man is rational, an econometrical model is an outcome to produce such knowledge.

7.3 Recommendations

The main result is that renewal is high when volatility is low and the time till expiration is short. Furthermore, the model assumes that the tenants are completely rational and only renew if there is a financial gain. If, by further research, the results will be tested in practice, it would be easier to interpret and to use in practice. A follow-up study could test if the height of volatility is of influence on the renewal rate of tenants. Also, it would be interesting to investigate whether a tenant has the same motives as a “homo economicus”.

As stated before, this model assumes the tenants' only motive is financial gain or loss. If a new research would conclude that financial motives are the main, or one of the main factors for tenants to renew, this model has a good angle. Therefore, the results can give a good indication. Thus, a follow-up research could reject or confirm the findings of this research.

So, to use this model in practice, further research is essential to test the model and the findings of the cases in practice. If one would use this model in practice, further research is recommended. In particular, the results and conclusions of this research could be tested in practice. If it turns out that there is a strong relationship in practice between a low volatility and a high renewal rate, the model gives an indication of the risk that is present. But, if the model says the exercise possibility is high, keep in mind that it is still a possibility. Therefore, it never gives any guarantees.

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Appendix

Appendix 1 Survey

Appendix 2 Market rental value cases

Appendix 3 Volatility cases

Appendix 4 Time till expiration cases

Appendix 5 Volatility in combination with time till expiration cases

Appendix 6 Volatility in combination with expiration in 1 year cases

Appendix 1

Survey: Predicting the future market rental value

Dear Mr/Mrs,

On behalf of my master thesis I would like to know your professional opinion of the expected market rental value per m^2 per year and the possible incentives for the following objects.

The data will be used to determine the volatility of the market rental value. This volatility is one input parameter in a model that will quantify the possibility that a tenant will renew his contract and determine the value of this possibility.

We assume that at the dates that are mentioned the object is free of tenants.

Also, we assume that the vacancy rates of the relevant areas are lower in improved conditions. And that the vacancy rates are higher in declining market conditions. In similar conditions they are obviously the same as now. The way how you interpret this is all up to you. It will not influence the usability of the results.

The results will be used aggregated so they will be anonymously.

Thank you in advance for entering your forecasts,

Kind Regards,

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Crystal Tower. Location: Amsterdam, Sloterdijk

Expected rental income per m^2 per year and expected incentives of this object on 30-6-2013

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Expected rental income per m^2 per year and expected incentives of this object on 30-6-2014

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Expected rental income per m^2 per year and expected incentives of this object on 30-6-2015

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Ito Tower. Location: Amsterdam, South Axis

Expected rental income per m^2 per year and expected incentives of this object on 30-6-2013

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Expected rental income per m^2 per year and expected incentives of this object on 30-6-2014

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Expected rental income per m^2 per year and expected incentives of this object on 30-6-2015

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Oval Tower. Location: Amsterdam, South-East

Expected rental income per m² per year and expected incentives of this object on 30-6-2013

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Expected rental income per m² per year and expected incentives of this object on 30-6-2014

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Expected rental income per m² per year and expected incentives of this object on 30-6-2015

A	market conditions similar as now	€______	______%
B	improved market conditions	€______	______%
C	declined market conditions	€______	______%

Appendix 2 Market rental value cases

Value and exercise possibility with a market rental value of $100.000

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$ 100.000	per year

S	$ 426.913	
K	$ 471.346	
σ	10,00%	per year
t	5	year
Rf	2%	per year

d1	0,1162
d2	-0,1074

N(d1)	0,5463
N(d2)	0,4572

Value call option	**$ 38.196**
Exercise possibility	**45,7%**

t	cash-flow	D.V	
Present value		$ 426.913	= S
6	$ 100.000	$ 88.797	
7	$ 100.000	$ 87.056	
8	$ 100.000	$ 85.349	
9	$ 100.000	$ 83.676	
10	$ 100.000	$ 82.035	

t	cash-flow	D.V	
Present value		$ 471.346	= K
1	$ 100.000	$ 98.039	
2	$ 100.000	$ 96.117	
3	$ 100.000	$ 94.232	
4	$ 100.000	$ 92.385	
5	$ 100.000	$ 90.573	

Value and exercise possibility with a market rental value of $200.000

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$ 200.000	per year

S	$ 853.825	
K	$ 942.692	
σ	10,00%	per year
t	5	year
Rf	2%	per year

d1	0,1162
d2	-0,1074

N(d1)	0,5463
N(d2)	0,4572

Value call option	**$ 76.392**
Exercise possibility	**45,7%**

t	cash-flow	D.V	
Present value		$ 853.825	= S
6	$ 200.000	$ 177.594	
7	$ 200.000	$ 174.112	
8	$ 200.000	$ 170.698	
9	$ 200.000	$ 167.351	
10	$ 200.000	$ 164.070	

t	cash-flow	D.V	
Present value		$ 942.692	= K
1	$ 200.000	$ 196.078	
2	$ 200.000	$ 192.234	
3	$ 200.000	$ 188.464	
4	$ 200.000	$ 184.769	
5	$ 200.000	$ 181.146	

Value and exercise possibility with a market rental value of $33.333

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$ 33.333	per year

S	$ 142.303	
K	$ 157.114	
σ	10,00%	per year
t	5	year
Rf	2%	per year

d1	0,1162
d2	-0,1074

N(d1)	0,5463
N(d2)	0,4572

Value call option	**$ 12.732**
Exercise possibility	**45,7%**

t	cash-flow	D.V	
Present value		$ 142.303	= S
6	$ 33.333	$ 29.599	
7	$ 33.333	$ 29.018	
8	$ 33.333	$ 28.449	
9	$ 33.333	$ 27.892	
10	$ 33.333	$ 27.345	

t	cash-flow	D.V	
Present value		$ 157.114	= K
1	$ 33.333	$ 32.679	
2	$ 33.333	$ 32.039	
3	$ 33.333	$ 31.410	
4	$ 33.333	$ 30.795	
5	$ 33.333	$ 30.191	

Appendix 3 Volatility cases

Value and exercise possibility with a volatility of 1%

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	426.913	
K	$	471.346	
σ	1,00%		per year
t	5		year
Rf	2%		per year

d1	0,0553
d2	0,0330

N(d1)	0,5221
N(d2)	0,5131

Value call option $ 4.021

Exercise possibility 51,3%

t		cash-flow		D.V	
Present value			$	426.913	= S
6	$	100.000	$	88.797	
7	$	100.000	$	87.056	
8	$	100.000	$	85.349	
9	$	100.000	$	83.676	
10	$	100.000	$	82.035	

t		cash-flow		D.V	
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Value and exercise possibility with a volatility of 10%

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	426.913	
K	$	471.346	
σ	10,00%		per year
t	5		year
Rf	2%		per year

d1	0,1162
d2	-0,1074

N(d1)	0,5463
N(d2)	0,4572

Value call option $ 38.196

Exercise possibility 45,7%

t		cash-flow		D.V	
Present value			$	426.913	= S
6	$	100.000	$	88.797	
7	$	100.000	$	87.056	
8	$	100.000	$	85.349	
9	$	100.000	$	83.676	
10	$	100.000	$	82.035	

t		cash-flow		D.V	
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Value and exercise possibility with a volatility of 20%

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	426.913	
K	$	471.346	
σ	20,00%		per year
t	5		year
Rf	2%		per year

d1	0,2258
d2	-0,2214

N(d1)	0,5893
N(d2)	0,4124

Value call option $ 75.710

Exercise possibility 41,2%

t		cash-flow		D.V	
Present value			$	426.913	= S
6	$	100.000	$	88.797	
7	$	100.000	$	87.056	
8	$	100.000	$	85.349	
9	$	100.000	$	83.676	
10	$	100.000	$	82.035	

t		cash-flow		D.V	
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Appendix 4 Time till expiration cases

Value and exercise possibility with expiration in 5 years.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	426.913	
K	$	471.346	
σ		10,00%	per year
t		5	year
Rf		2%	per year

d1	0,1162
d2	-0,1074

N(d1)	0,5463
N(d2)	0,4572

Value call option	$	38.196

Exercise possibility	45,7%

t		cash-flow		D.V	
Present value			$	426.913	= S
6	$	100.000	$	88.797	
7	$	100.000	$	87.056	
8	$	100.000	$	85.349	
9	$	100.000	$	83.676	
10	$	100.000	$	82.035	

t		cash-flow		D.V	
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Value and exercise possibility with expiration in 3 years.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	444.160	
K	$	471.346	
σ		10,00%	per year
t		5	year
Rf		2%	per year

d1	0,2933
d2	0,0697

N(d1)	0,6154
N(d2)	0,5278

Value call option	$	48.221

Exercise possibility	52,8%

t		cash-flow		D.V	
Present value			$	444.160	= S
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	
6	$	100.000	$	88.797	
7	$	100.000	$	87.056	
8	$	100.000	$	85.349	

t		cash-flow		D.V	
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Value and exercise possibility with expiration in 1 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	462.104	
K	$	471.346	
σ		10,00%	per year
t		5	year
Rf		2%	per year

d1	0,4705
d2	0,2469

N(d1)	0,6810
N(d2)	0,5975

Value call option	$	59.863

Exercise possibility	59,7%

t		cash-flow		D.V	
Present value			$	462.104	= S
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	
6	$	100.000	$	88.797	

t		cash-flow		D.V	
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Appendix 5 Volatility in combination with time till expiration cases

Value and exercise possibility with a volatility of 10% and expiration in 5 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year
S	$	426.913	
K	$	471.346	
σ	10,00%		per year
t	5		year
Rf	2%		per year
d1	0,1162		
d2	-0,1074		
N(d1)	0,5463		
N(d2)	0,4572		
Value call option	$	38.196	
Exercise possibility	45,7%		

t	cash-flow	D.V	
Present value		$ 426.913	= S
6	$ 100.000	$ 88.797	
7	$ 100.000	$ 87.056	
8	$ 100.000	$ 85.349	
9	$ 100.000	$ 83.676	
10	$ 100.000	$ 82.035	

t	cash-flow	D.V	
Present value		$ 471.346	= K
1	$ 100.000	$ 98.039	
2	$ 100.000	$ 96.117	
3	$ 100.000	$ 94.232	
4	$ 100.000	$ 92.385	
5	$ 100.000	$ 90.573	

Value and exercise possibility with a volatility of 6% and expiration in 3 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year
S	$	444.160	
K	$	471.346	
σ	6,00%		per year
t	5		year
Rf	2%		per year
d1	0,3696		
d2	0,2355		
N(d1)	0,6442		
N(d2)	0,5931		
Value call option	$	33.173	
Exercise possibility	59,3%		

t	cash-flow	D.V	
Present value		$ 444.160	= S
4	$ 100.000	$ 92.385	
5	$ 100.000	$ 90.573	
6	$ 100.000	$ 88.797	
7	$ 100.000	$ 87.056	
8	$ 100.000	$ 85.349	

t	cash-flow	D.V	
Present value		$ 471.346	= K
1	$ 100.000	$ 98.039	
2	$ 100.000	$ 96.117	
3	$ 100.000	$ 94.232	
4	$ 100.000	$ 92.385	
5	$ 100.000	$ 90.573	

Value and exercise possibility with a volatility of 2% and expiration in 1 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year
S	$	462.104	
K	$	471.346	
σ	2,00%		per year
t	5		year
Rf	2%		per year
d1	1,8156		
d2	1,7709		
N(d1)	0,9653		
N(d2)	0,9617		
Value call option	$	35.901	
Exercise possibility	96,2%		

t	cash-flow	D.V	
Present value		$ 462.104	= S
2	$ 100.000	$ 96.117	
3	$ 100.000	$ 94.232	
4	$ 100.000	$ 92.385	
5	$ 100.000	$ 90.573	
6	$ 100.000	$ 88.797	

t	cash-flow	D.V	
Present value		$ 471.346	= K
1	$ 100.000	$ 98.039	
2	$ 100.000	$ 96.117	
3	$ 100.000	$ 94.232	
4	$ 100.000	$ 92.385	
5	$ 100.000	$ 90.573	

Appendix 6 Volatility in combination with expiration in 1 year cases

Value and exercise possibility with a volatility of 10% and expiration in 1 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	462.104	
K	$	471.346	
σ	10,00%		per year
t	5		year
Rf	2%		per year

d1	0,4705
d2	0,2469

N(d1)	0,6810
N(d2)	0,5975

Value call option	$	59.863

Exercise possibility	59,7%

t	cash-flow		D.V		
Present value			$	462.104	= S
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	
6	$	100.000	$	88.797	

t	cash-flow		D.V		
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Value and exercise possibility with a volatility of 6% and expiration in 1 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	462.104	
K	$	471.346	
σ	6,00%		per year
t	5		year
Rf	2%		per year

d1	0,6648
d2	0,5307

N(d1)	0,7469
N(d2)	0,7022

Value call option	$	45.683

Exercise possibility	70,2%

t	cash-flow		D.V		
Present value			$	462.104	= S
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	
6	$	100.000	$	88.797	

t	cash-flow		D.V		
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	

Value and exercise possibility with a volatility of 2% and expiration in 1 year.

Black - Scholes Real Option model for real estate lease contracts

Rental market value per year	$	100.000	per year

S	$	462.104	
K	$	471.346	
σ	2,00%		per year
t	5		year
Rf	2%		per year

d1	1,8156
d2	1,7709

N(d1)	0,9653
N(d2)	0,9617

Value call option	$	35.901

Exercise possibility	96,2%

t	cash-flow		D.V		
Present value			$	462.104	= S
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	
6	$	100.000	$	88.797	

t	cash-flow		D.V		
Present value			$	471.346	= K
1	$	100.000	$	98.039	
2	$	100.000	$	96.117	
3	$	100.000	$	94.232	
4	$	100.000	$	92.385	
5	$	100.000	$	90.573	