University of Groningen
Msc. in Population Studies

Jesus Adrian Alvarez Martinez
Student Number: S2835525
j.a.alvarez.martinez@student.rug.nl

Title: Modeling the Fertility Transition: Incorporating the effects of mortality on fertility.

Abstract

The current fertility projections of the United Nations (2013b) indicate that in the long term all countries in the world will end with fertility levels close to two children per woman. In recent years, most of the western countries have already attained low fertility levels; however, there are some other countries in which the uncertainty of reaching those levels is still high, for instance some African sub-Saharan and Asian countries.

In this research I analyzed regularities on fertility and child mortality of 63 countries around the world in order to refine the fertility projection of a group of countries that nowadays show high fertility levels. In this regard, a Bayesian approach is used and when child mortality is incorporated into the model, the slope of the fertility trajectory changes by slowing the pace of the decline and postponing the convergence to replacement levels.

Key words: Fertility transition, mortality, Bayesian approach, sub-Saharan Africa.

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1. Introduction

1.1. Problem Statement

According to the forecast of the United Nations (2013b) global fertility will decline to 2.24 children per woman in the period from 2045 to 2050 and 1.99 children per woman in the period from 2095 to 2100. Nowadays, the Western countries are not so far from that forecast, however, there are some other countries in which the uncertainty of reaching low fertility levels is high because for the last years they have not shown a drop in the fertility levels. Most of the countries in this context are developing countries, for instance African sub-Saharan countries. Moreover, if fertility achieves the levels mentioned above, the world population would reach 10.9 billion inhabitants by 2050 and 16.6 billion by the end of the century. Nevertheless, slightly changes in fertility trends will bring a great impact on total population size; a fertility path half a child below would lead to a population of 8.3 billion by 2050 and 6.8 billion by the end of the 21st century (United Nations, 2013b).

Hereof, counting with an overview of the future possible fertility scenarios is fundamental. Additionally, many studies have stated that fertility is closely linked to the mortality trends and these findings could be incorporated to generate future fertility trajectories.

1.2. Objective

The objective of the research is to analyze the effects of infant mortality on fertility for a set of countries that are in the third stage of the demographic transition in order to develop a model to better estimate future fertility levels for a group of countries that are in the second stage of the demographic transition.

1.3. Research Questions

How does the experience of countries that are in the third stage of the Demographic Transition refine the estimation of future fertility levels of a set of countries that are in the second stage of the Demographic Transition?

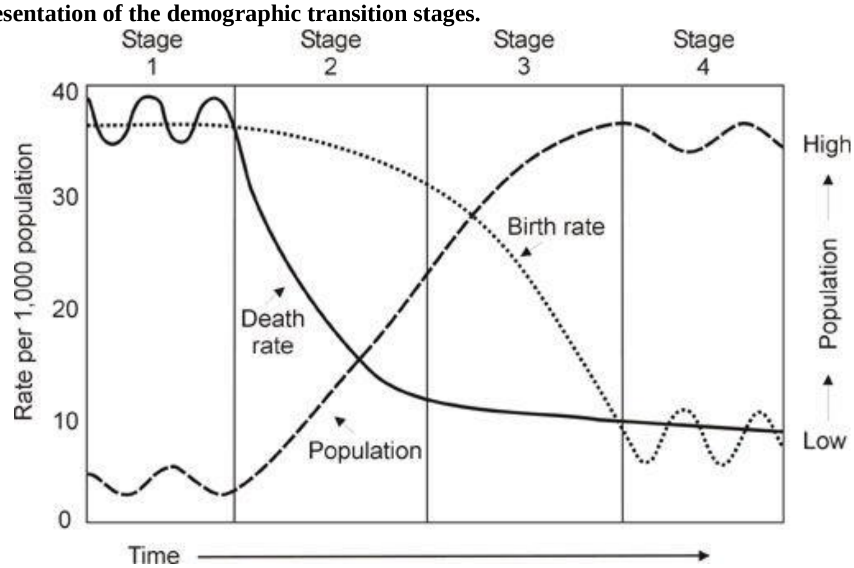
a. How the infant mortality is associated with the fertility trends in a set of countries that are in the third stage of the Demographic Transition?
b. How the fertility of a set of countries that are in the second stage of the Demographic Transition will develop for the next 50 years?

2. Theoretical framework

2.1. Demographic Transition theory

This research is based on the Demographic Transition theory. This theory was originally developed by Warren Thompson in 1929 and describes the changes that take place in populations over the time. The model is divided in four stages depending on the mortality and fertility levels of the analyzed society (Weeks, 2008). The first stage is portrayed by pre-industrial societies whose have high growth population potential because of its high birth and death rates. The second stage is the transition from high to low death rates. As birth rates remain high, the population starts to grow rapidly. The third stage is a time when death rates are as low as there are likely to go, while fertility may continue to decline to the point that the population might eventually decline in numbers. Finally, the last stage is characterized by low birth and death rates and the population growth is imperceptible or even enters a decline and in consequence it creates an economic constraint as well as a decline in the total population.

Graph 1. Representation of the demographic transition stages.



Note: The graph was taken from the article Demographic Transition Theory by Dudley Kirk (1996).

Several years after the first approach presented by Warren Thompson, the demographic transition theory was reformulated mainly by the European Fertility Project, the Easterlin hypothesis and was portrayed as a set of interrelated transitions (Kirk, 1996). Taken together they help us to better understand the causes and the consequences of the population changes. On the one hand, we can point out that the first transition to occur the health and mortality transition which is described by the shift from deaths at younger ages due to communicable diseases to deaths at older ages due to degenerative diseases. This transition begins with a decline in mortality due to changes in society that improve health and ability to resist disease and prevent premature death (Weeks, 2008). This transition is followed by the fertility transition which outlines the shift from natural to controlled fertility over the time. This transition begins with a decline in mortality leading to a greater survival of children which motivates people to think about limiting the number of children they are having and in consequence a following decline in fertility.

It is important to remark that the Demographic Transition theory is based on the experience of the developed nations and it takes into account the economic and social development. One of the regularities observed in those countries refers to a first decline in mortality and then after some time lag, to a decline in fertility (Kirk, 1996).

2.2. Literature review

2.2.1. An overview of the fertility trends in recent years

United Nations (2013b) considers a country that have low fertility if it has a Total Fertility Rate (TFR) around 2.0 children per woman or less, furthermore, a high-fertility country is that one that has a TFR above 3.2 children per woman. Most low-fertility countries all around the globe, that already count with a TFR less than 1.5 children per women during the period 2005-2010 are projected to either increase their total fertility levels by 0.3 children per woman or remain stable as at 2035 (United Nations, 2014). These type of countries are becoming more diverse geographically, including many more countries in Asia, Latin America and the Caribbean.

Fertility on high-fertility countries is expected to decrease in most of them. Some Asian countries will experience an abrupt decline, whereas in other African countries such as Mali and Niger the fertility decline is projected to be more gradual (United Nations, 2012). Most of these countries are concentrated in sub-Saharan Africa and according to some studies there have been important declines in adolescent fertility but in general adolescent fertility remains very high in most of the the sub-Saharan Africa, particularly in Middle Africa and Western Africa (Sneeringer, 2009). Regarding these countries, the United Nations (2014) projection states that they will reach total fertility of 2.6 children per woman or below as at 2050.

According to the United Nations (2013b) forecast on fertility all of the developing countries will finish their transitions by 2030 reaching levels at or below 2.2 children per woman; the only exceptions will be sub-Saharan Africa and some West Asia countries, where the total fertility is expected to be between 3.5 and 2.8 children per woman. The outcome of these projections will change if the fertility declines slow down or if it changes the speed of decline. Some researchers claim attention of this issue, for instance, Bongaarts (2008) analyzed past fertility trends in the developing world based on the United Nations estimates of TFRs of 143 “less developed countries” including all countries in Africa, Asia, and Latin America and the Caribbean and he found that the speed of the fertility decline for countries in early phases of the demographic transition proceed relatively rapidly at the beginning of the transition, but is gradually slowed down at lower TFR levels and moreover, he argues that these regularities could be repeated on sub-Saharan and Asian countries that nowadays present high TFRs. In addition to this, some other researchers have analyzed the speed of the fertility decline on some developing countries, particularly Bakole (2010) argued a decline in the desired family size in Nigeria during recent years, pointing out that this observation is an important underlying force that pushes the fertility transition but notwithstanding this finding, he stated that the speed of the fertility decline in that country that the United Nations projections seems to be too rapid and unrealistic in the face of the current socio-economic and political environment of the country. These findings together, give us a clue that those projections should be revised.

2.2.2. An overview of the mortality levels in recent years

Life expectancy at birth summarizes mortality rates in a population across all ages for a given time period. It expresses the average number of years a person would live if he or she were exposed throughout his or her lifetime to the age-specific mortality risks of that period. According to the United Nations mortality report (2013a) during the early 1950s, global life expectancy stood at 46.9 years and during the 1990s a major progress in survival had already been achieved, reflected in the global life expectancy at birth of 64.8 years in the period from 1990 to 1995. Since then, another 5.2 years of life expectancy have been attained, and global life expectancy is estimated to have reached 70 years in the period from 2010 to 2015. These observed trends could be related with the entrance of most of the countries to the third and fourth stage of the demographic transition. Nevertheless, and in order to study mortality patterns during the life cycle we have to look into the different ages of a living person, for instance the indicator of development and the well-being of a child. This phenomenon is studied with the under-five mortality, which is expressed as the probability of dying between birth and the exact age of five. During the period from 1950 to 1955, 21 per cent of all children born worldwide did not reach their fifth birthday and by the period from 2005 to 2010 this rate had fallen to 59 deaths per 1,000 births. However, this rate in least developed regions still remains at a relatively high level with around 112 deaths per 1,000 births in the period from 2005 to 2010. Moreover, during the period from 1990 to 1995, close to half of births worldwide took place in countries with under-five mortality rates greater than 100 deaths per 1,000 live births but nowadays less than 1 in 10 babies in countries with under-five mortality rates below 45 deaths per 1,000 live births. Despite the achievements in reducing infant mortality and under-five mortality, the global averages for these indicates held back in particular by high levels of infant and child mortality still prevailing in parts of Africa (United Nations, 2013b). Actually, Africa has shown the highest level of infant mortality in 2010-2015 at 64 deaths per 1,000. In seven countries located in Middle and Western Africa, under five mortality is estimated to be greater than 150 deaths per 1,000 live births during the period 2010 to 2014 which is more than 50 times higher than in the world's populations with the lowest mortality levels.

During early 1990s, trends in mortality risks between ages 15 and 60 were relatively flat at the global level, but improvements in adult survival accelerated between the period 2000 to 2005 and the period from 2010 to 2015 such that the global probability of dying in that age range declined. In this context, Africa presents the higher probability of dying between ages 15 and 60 with 296 deaths per 1,000 (Bankole, 2010). Also, if we look at the mortality of people over 60 we can appreciate that since the early 1990s, not only the number of people surviving to age 60 has increased, but they are living longer into old age as well. Worldwide, the life expectancy at age 60 increased from 18 years in the period from 1990 to 1995 to 20 years in the period from 2010 to 2015 (United Nations, 2013a). Particularly the African countries' life expectancy at age 60 reached 16.4 which represent the lowest life expectancy in the world followed by Asia with 19.3 years and Latin America and the Caribbean with 21.9 years.

2.2.3. Mortality effects on fertility

In most of the countries that are now in advanced stages of the demographic transition was observed that child mortality tends to decrease first and then the fertility follows it. This lag gives us a clue that child mortality has an aftermath on fertility (Angeles, 2010). The relevance of this observation lays on the possibility of modeling this relationship in order to estimate the future fertility trends based on the link with their mortality.

We can mention the “unified growth theories” as an antecedent of the study of the role that the demographic transition plays in the transition of pre-industrial times and the industrial revolution as cause and effect of the transition in growth regimes. Additionally, Gary Becker (1969) developed his work related to the fertility transitions based in the assumption of the existence of the parental utility which is a function related to the number of children (child quantity) and the education or human capital that children would have (child quality). In this context, fertility and children’s human capital were introduced as variables of the child quality and child quantity. The link of this theory with mortality came because high mortality makes investment in children’s human capital less attractive, so as long as mortality declines, parents would substitute child quality for child quantity. In addition, there are some other factors that affect the decision of prefer child quality rather than child quantity, for instance the economic and technological development. In this context parents will prefer to invest in children education because it increases the children’s human capital (Angeles, 2010). Moreover, some demographers have remarked that the effects of the economic development and education produce a decrease in the fertility levels. In his study, Lorentzen (2008) argues that mortality may affect economic growth through several channels and fertility is one of them. So he applies a regression measure of the fertility versus the adult mortality rate and the infant mortality rate (IMR), the results of this analysis provide support that both mortality rates have significant impact on fertility. Also, Acemoglu and Jonhnson (2007) found in their study that life expectancy is strongly linked with the total population as well as the total number of births.

Many researches have been performed in order to better understand the relationship among mortality and fertility, for instance, Angeles (2010) studied the role of mortality as a driver for fertility change over 118 countries over the period 1960-2000 and he showed that changes in mortality have a large impact on fertility reductions and can account for a major part of the fertility change characterized on the demographic transitions. Also, Caudell (2015) examined the association between within-country changes in economics, mortality, and female empowerment and the onset and output of female reproduction within 167 countries across a 40-year period, from 1970 to 2010. In general, he found that fertility rates were positively associated with mortality rates and negatively associated with economic development and advances in female empowerment. Despite of this, the relationship was not linear across all levels of the three factors. Higher levels of population mortality and economic development were associated with decreasing impacts on fertility rates.

Taking into account the findings of the researches mentioned above, there is sufficient empirical evidence of the existence correlation between the behavior of mortality and fertility. In this regard, I will use past child mortality trends in order to better predict the future fertility trajectories among the countries of study.

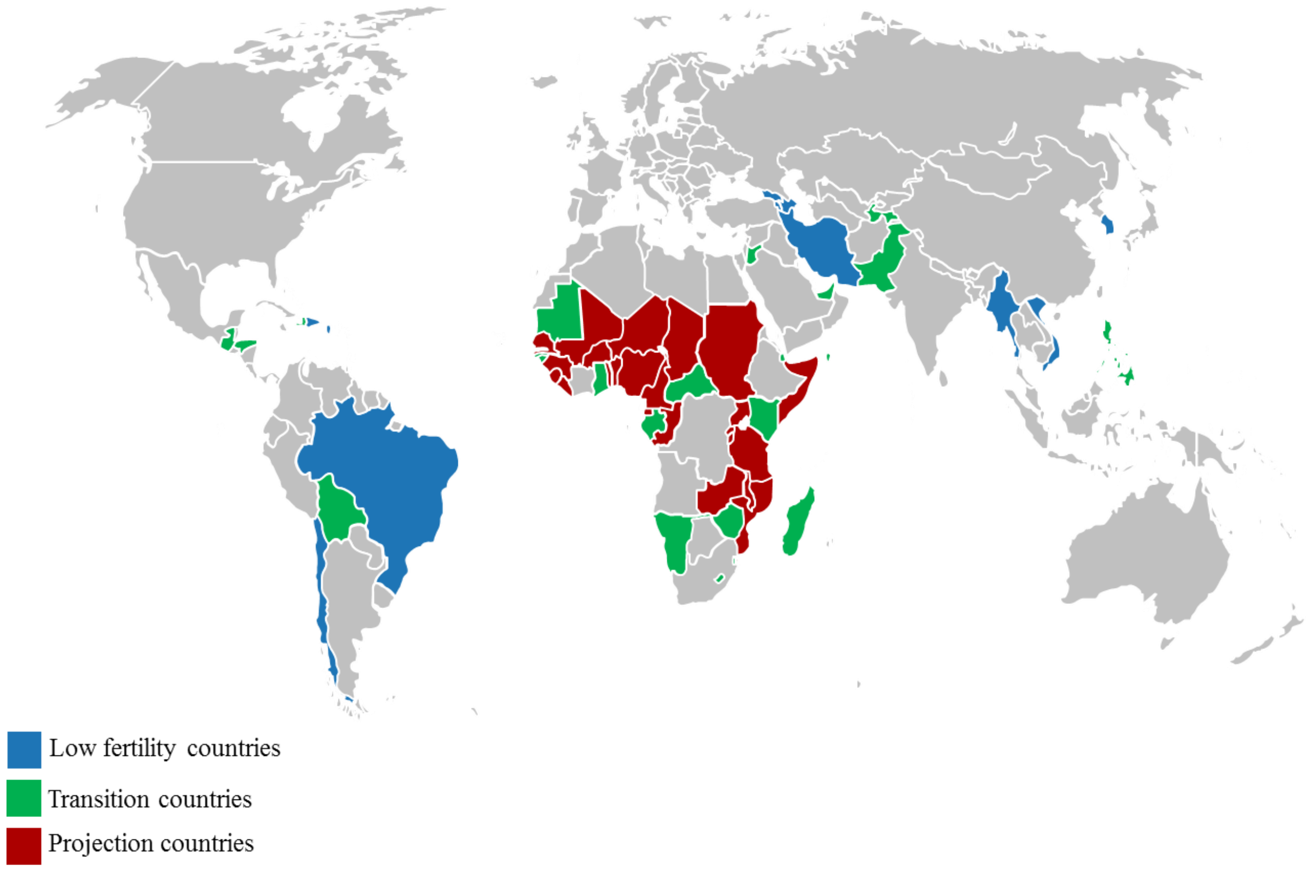
2.2.4. Countries of Study

The selection criteria of the countries to be included in the study lays in their fertility levels during the period 2005-2010 which is in line with the criteria used by the United Nations' World Population Prospects (2013), so the countries were grouped in three sets according to their geographical location and total fertility rate. First, low fertility countries are those having total fertility rates between 1.2 and 2.0 children per woman, and they are represented for eight Asian Countries and seven Latin America & Caribbean countries. According to their characteristics, these countries are supposed to be in the third stage of the Demographic Transition. Secondly, I split the high fertility countries in two sets: transition and projection countries. The set of "transition countries" shows a total fertility rate between 2.0 and 5.0 children per women and it is represented by five countries from Latin America & Caribbean, five countries from Asia and fifteen countries from Africa. This set of countries portrays the transition stage in which the fertility has dropped from high to low levels. Finally, the set of "projection countries" includes twenty-three countries from Africa that have a total fertility rate greater than 5.0 children per woman and they have not presented yet a drop on fertility during the past years so they are situated in the second stage of the Demographic Transition. The reason of splitting the countries in three sets is because the interest of projecting fertility on those countries that are still having more than 5.0 children per women by incorporating the information of the low fertility and transition countries.

Table 1. List of countries included in the study.

Set	Countries	Characteristic
Low-fertility countries	Asia: *Azerbaijan, Lebanon, Republic of Korea, Viet Nam, Iran, Georgia, Armenia, Thailand.* Latin America and the Caribbean: *Saint Lucia, Bahamas, Brasil, Chile, Barbados, Trinidad y Tobago, Aruba.*	*TFR between 1.2 and 2.0 children per woman in the last observed period.*
Transition countries	Latin America and the Caribbean: *Honduras, Bolivia, Haiti, Guatemala, Curacao.* Asia: *Phillipines, Jordan, Tajikistan, Pakistan, United Arab Emirates.* Africa: *Central African Republic, Comoros, Djibouti, Gabon, The Gambia, Ghana, Guinea-Bissau, Kenya, Lesotho, Madagascar, Mauritania, Namibia, Sudan, Swaziland, Zimbawe.*	*TFR between 2.0 and 5.0 children per woman in the last observed period.*
Projection countries	Africa: *Niger, Mali, Burkina Faso, Nigeria, Benin, Liberia, Sierra Leone, Senegal, Chad, Congo, Guinea, Cameroon, Somalia, Burundi, Uganda, Zambia, Malawi, Mozambique, Tanzania, South Sudan, Sao Tome and Principe, Togo, Rwanda.*	*TFR more than 5.0 children per woman in the last observed period.*

Graph 2. Map of the selected countries split by their fertility levels observed in 2014

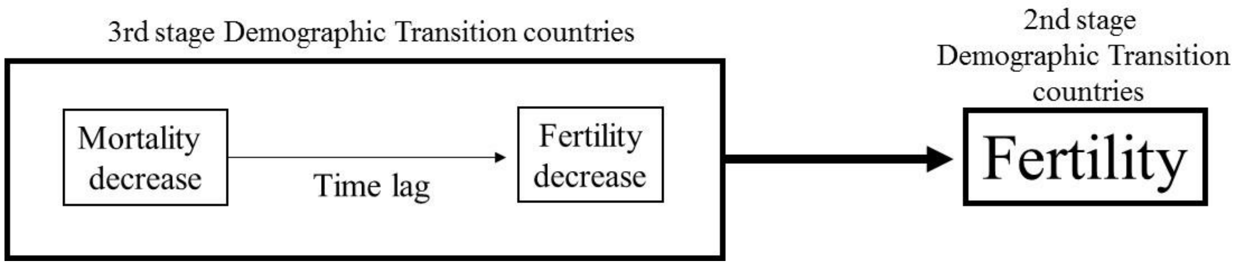


Source: Self-computed with information retrieved of the US Bureau Census (2015)

2.2.5. Conceptual Model

The following is a graphical representation of the relationship between the key concepts of the study.

Graph 3. Conceptual model



2.2.6. Hypothesis

The following statements are presented as the hypothesis to be proven by the empirical study concerned in this research.

a) Fertility of the 3rd stage country is related to the drop of mortality of the same country when it was on a 2nd stage.

$$Fertility^{A}_{t+1} = f(Mortality^{A}_{t})\ ?$$

b) Fertility of a 3rd stage country can be extrapolated to the fertility of a 2nd stage country at time t+1.

$$Fertility^{B}_{t+1} = f(Fertility^{A}_{t})\ ?$$

Graph 4. Relationship of the stages of the demographic transition

Note: The graphs were computed in order to show the relationship of fertility and mortality in different stages of the Demographic Transition.

3. Data and methods

This research is based on quantitative methods in which a systematic investigation will be performed. Empirical data will be analyzed with the use of statistical methods as well as a probabilistic approach. First, I will analyze the collected demographic data and its description of statistical measures. Secondly, I will perform statistical tests to prove the correlation among the variables in the model and in a final stage I will make forecasts of the fertility levels for the next 50 years using Bayesian probabilistic inference in order incorporate more information to the model and ends up with a probabilistic density function that allow us to visualize the future fertility levels of the previous selected countries.

3.1. Methods

3.1.1. Bayesian Approach

Bayesian inference is defined by Gelman et al (2013) as the process of fitting a probability model to a set of data and summarizing the result by a probability distribution on the parameters of the model and with unobserved quantities such as predictions for new observations. The first reason for using a Bayesian approach is that it facilitates a common-sense interpretation of statistical conclusions, for instance a Bayesian projection interval for an unknown quantity of interest can be directly regarded as having a high probability of containing the unknown quantity, additionally, Bayesian methods enable statements to be made about data and based on the partial knowledge available concerning some situation unobservable or as yet unobserved in a systematic way, using probability as a benchmark, so the guiding principle is that the state of knowledge about anything unknown is described by a probability distribution.

The relevance of using a Bayesian approach in making fertility forecasts relies on the fact that the projections could be refined while including uncertainty and more information to the probability distribution. This process could be divided into three main steps. First, set up a full probability model, which consist in a joint probability distribution for all observable and unobservable quantities in a problem. The model should be consistent with knowledge about the underlying scientific problem and the data collection process. The second step is that given the data, calculate and interpret a posterior distribution of the unobserved quantities of ultimate interest and the last step refers to evaluation of the fit of the model and the implications of the resulting posterior distribution. The statistical model introduced by Alkema (2011) and used in this research, incorporates a Bayesian Hierarchical modeling, which are used when information is available on several different levels of observational units, in the next section I will explain more in detail such model.

3.1.2. Statistical model

The model used to project the fertility transition in the selected countries will be the one introduced by Alkema (2011). As of 2013, this model is used by the United Nations in their probabilistic projections and its main advantage is that it is based on a Bayesian approach, which, as I mentioned in the previous section, allows incorporating more information a priori in order to better fit the data and, in consequence, refine the projections. Moreover, this approach incorporates uncertainty in the forecast of the TFR by constructing a probability density function of all the possible outcomes.

The model works in two main steps. In the first step, the model uses the information of all the selected countries in order to create a global distribution of the set of parameters (θ) of the double logistic function ($g(\theta, f_{c,t})$) that will define the general shape of the decline. In the second step, the distribution of the global decline function, just created in the first step, is combined with the specific characteristics of the country and then used to simulate several TFR trajectories using the Monte Carlo Markov Chain method for all the twenty-three "projection countries", and afterwards, the median of all the simulated trajectories for each country is drawn and selected as the best estimate of the future TFR. This model is called a Bayesian Hierarchical model since it incorporates different levels or hierarchies of information; the first level is given by the global distribution of all the selected countries ("a priori" distribution) and the second level arises with the country-specific distribution given by the particularities of the county to be projected ("a posteriori" distribution).

3.1.3. Modeling the fertility transition

Let $f_{c,t}$ be the TFR of the country *c* at time *t*, the main equation of the model is given by:

$$f_{c,t+1} = f_{c,t} - r_{c,t} + \varepsilon_{c,t}$$

Where $r_{c,t}$ is the expected 5-year decrement and $\varepsilon_{c,t}$ is a random distortion term. The model takes into account the behavior of the TFR on a 5-year period modeling the involved decrements as follows:

$$r_{c,t} = \begin{cases} g(\theta, f_{c,t}); & f_{c,t} > 1 \\ 0; & otherwise \end{cases}$$

$$\varepsilon_{c,t} = \begin{cases} N(m_t, s_t^2); & t = \tau_c \\ N\left(0, \sigma(f_{c,t})^2\right); & otherwise \end{cases}$$

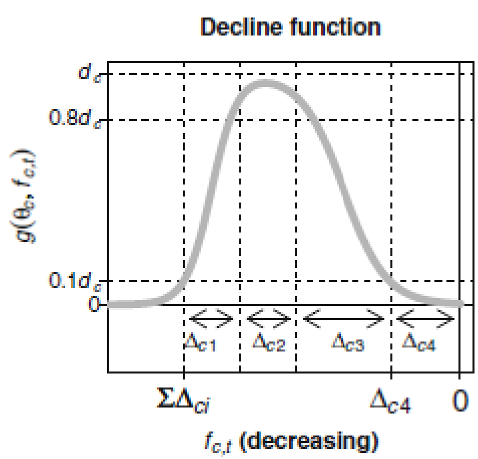
τ_c : Start period of the fertility transition of the country *c*.

The expected 5-year decrement ($d_{c,t}$) of the TFR is given by double logistic function $g(\theta, f_{c,t})$,

$$g(\theta, f_{c,t}) = \frac{-d_c}{1 + exp\left(\frac{-2ln(9)}{\Delta c_1}(f_{c,t} - \sum_i \Delta c_i - 0{,}5\Delta c_i\right)} + \frac{d_c}{1 + exp\left(\frac{-2ln(9)}{\Delta c_3}(f_{c,t} - \Delta c_4 - 0.5\Delta c_3\right)}$$

$\theta = (\Delta c_1, \Delta c_2, \Delta c_3, \Delta c_4, d_c)$
$\Delta c_i =$ Decline parameters of the TFR
$U_c = \sum_i \Delta c_i$: Start period $f_{c,t}$
d_c : Maximum possible 5-year decrement of the country c

Graph 5. Representation of the fertility decline function



Source: Alkema et al. (2011)

The estimation of the hyperparameters $\theta = (\Delta c_1, \Delta c_2, \Delta c_3, \Delta c_4, d_c)$ and their distributions (hyperpriors) is given by a Bayesian hierarchical model, which assumes that for all countries each of these parameters are drawn from a probability distribution that represents the range of outcomes of that parameter across all the countries. For a specific parameter in a specific country the probability distribution based on the third stage countries experience is then updated using the bayes's theorem with the observed declines in the country, which results in the posterior distribution of the parameter of interest**.** By including two levels of hierarchy in the Bayesian hierarchical model, the projected fertility decline for each country depends on its own past trends and the experience of the third-stage countries. A Markov chain Monte Carlo (MCMC) algorithm is used to derive samples of the posterior distributions of the parameters of the fertility transition model. The results are based on a run of 5 MCMC chains with 10,000 iterations each one, in which the first 2,000 are discarded, yielding 8,000 samples per MCMC from the posterior distributions of all model parameters.

The predictive distribution is then represented by a sample $\{f_{c,t+1}{}^{(i)}: i = 1,2,\dots,I\}$. The i-th member of the sample $f_{c,t+1}{}^{(i)}$ is given by $f_{c,t+1}{}^{(i)} = f_{c,t}{}^{(i)} - r_{c,t}{}^{(i)} + \varepsilon_{c,t};\ i = 1,2,\dots,I$ where $f_{c,t+1}{}^{(i)}$ is the i-th member of the sample of TFR outcomes in period t, $r_{c,t}{}^{(i)}$ is the expected decrement given by the decline function evaluated at $f_{c,t}{}^{(i)}$ and $\theta_c{}^{(i)}$ (the i-th sample of parameter vector θ_c) and $\varepsilon_{c,t}$ is a random draw from a normal probability function $N\left(0, \sigma(f_{c,t}{}^{(i)})^2\right)$ with mean 0 and standard deviation $\sigma(f_{c,t}{}^{(i)})^2$. After a large sample of TFR trajectories is constructed, the "best" TFR projection is given by the median outcome of the TFR trajectories in each period and also the 95 percentile of the distribution is taken in order to determine the projection interval in which the TFR could move.

3.1.4. Incorporating the effect of mortality to the estimation

According to the study conducted by Angeles (2010) and the researches mentioned in the theoretical review, there is empirical evidence that there is a time lag between a decrease on the IMR and a drop on the TFR so in order to incorporate the effect of the infant mortality on fertility, some variables are introduced as follow. Let $i_{c,t}$ be the infant mortality rate of the country c at time t, then the decrease of the mortality measure from one year to another is given by $m_{c,t} = \frac{i_{c,t}}{i_{c,t+1}} - 1$. Therefore, q_m represents the year where the maximum of all the observed values $m_c = \max\left(m_{c,t}, m_{c,t+1}, \dots, m_{c,t+n}, 0\right)$ is reached where $t+n$ is the most recent observed period. Analogously for the TFR, q_f represents the year in which the maximum decrease on fertility is achieved so assuming that $q_f \le q_m$ then the equation $q_f = q_m + l_c$ indicates the relationship of both time series, and the variable l_c is introduced as a measure of the time lag between the beginning of the mortality transition and the fertility transition of a given country c.

The equation $q_f = q_m + l_c$ is then estimated using least squares linear regression with information of the selected low fertility and transition countries described on the section 2.2.4. The results are indicated in the table 2.

Table 2. Results of the least squares regression on the parameter q_f

Observations	R Square	Adj. R Square
40	0.241	0.221

	Coefficients	P-value
Intercept (l_c)	1,193.259*	0.000
Max decrease IMR (q_m)	0.401*	0.001

*p<0.05

The regression was performed with 40 observations of 17 low fertility countries and 23 transition countries. According to the R square tests, around 24% of the variation in the dependent variable q_f is explained by the variation in the independent variables (l_c , q_m). Moreover, we can appreciate that the coefficients have a positive effect on the variable q_m and that are statistically different from zero in a 95% significance level. In general terms, the model suits the data well and there is sufficient statistical evidence that the Equation 1 consider the existing relationship between the decrease on the IMR and the TFR; then the Equation 1 is used to calculate q_f^* on the projection countries and further estimation of the parameter ${\tau_c}^*$ by setting the five-year period corresponding to the estimated q_f^* and in which the fertility decline has started.

$$q_f^* = 0.401 q_m + 1{,}193.259 \qquad \textbf{(Equation 1)}$$

The results of the calculation mentioned above are shown in the Table 3.

Table 3. Results of the least squares regression on the parameter q_f

Country	q_m	τ_c *
Benin	1988	1985-1990
Burkina Faso	1988	1985-1990
Burundi	1992	1990-1995
Cameroon	1986	1985-1990
Chad	1990	1990-1995
Congo (Brazzaville)	1986	1985-1990
Guinea	1980	1980-1985
Liberia	1994	1990-1995
Malawi	1994	1990-1995
Mali	1994	1990-1995
Mozambique	1990	1990-1995
Niger	2001	2000-2005
Nigeria	1994	1990-1995
Rwanda	1992	1990-1995
Sao Tome and Principe	1994	1990-1995
Senegal	1986	1985-1990
Sierra Leone	1996	1995-2000
Somalia	1992	1990-1995
South Sudan	1998	1995-2000
Tanzania	1994	1990-1995
Togo	1988	1985-1990
Uganda	1988	1985-1990
Zambia	1994	1990-1995

3.1.5. Differences between the selection of the parameter τ_c and τ_c^*

The calculation of the parameter τ_c in the model by Alkema et al. (2011) is given by a deterministic rule, in which they identify the starting period of the fertility transition as the most recent period with a local maximum ($L_{c,t}$) within 0.5 child of the global maximum observed TFR (M_c), then, if this local maximum is above 5.5, the corresponding period is defined as the start period of the fertility transition; otherwise τ_c < 1950–1955.

$$\tau_c = \begin{cases} \max\{t: (M_c - L_{c,t}) < 0.5\}, & if\ \ L_{c,t} > 5.5; \\ < 1950 - 1955, & otherwise. \end{cases}$$ **(Equation 2)**

I replicated the Equation 2 in order to set the parameter τ_c for all the projection countries and compare them with the outcomes of the estimation of τ_c^*. In the Table 4 we can observe that in average τ_c is 3 observed 5-year period earlier than τ_c^*, however, in some countries the difference between both parameters is particularly higher, for instance in Cameroon, Mali Niger, Nigeria, Sao Tome and Principe and Uganda the difference is up to 8 5-year periods, so the model stated in Alkema et al (2011) indicates that the fertility transition in those countries has started in 1950. This statement differs to the findings on previous researches in the field, for instance Feyisetan et al (2002) stated that the fertility transition in Nigeria has started within the decade from 1990 to 2000.

Table 4. Comparison between the two approaches to setting up the initial period of fertility decline on the projection countries.

Country	τ_c	τ_c*	τ_c -τ_c*
Benin	1975-1980	1985-1990	-2
Burkina Faso	1975-1980	1985-1990	-2
Burundi	1975-1980	1990-1995	-3
Cameroon	1950-1955	1985-1990	-7
Chad	1995-2000	1990-1995	1
Congo (Brazzaville)	1970-1975	1985-1990	-3
Guinea	1960-1965	1980-1985	-4
Liberia	1995-2000	1990-1995	1
Malawi	1975-1980	1990-1995	-3
Mali	1950-1955	1990-1995	-8
Mozambique	1980-1985	1990-1995	-2
Niger	1950-1955	2000-2005	-10
Nigeria	1950-1955	1990-1995	-8
Rwanda	1975-1980	1990-1995	-3
Sao Tome and Principe	1950-1955	1990-1995	-8
Senegal	1975-1980	1985-1990	-2
Sierra Leone	1970-1975	1995-2000	-5
Somalia	2000-2005	1990-1995	2
South Sudan	2005-2010	1995-2000	2
Tanzania	1975-1980	1990-1995	-3
Togo	1980-1985	1985-1990	-1
Uganda	1950-1955	1985-1990	-7
Zambia	1980-1985	1990-1995	-2

The value in the column $\tau_c - \tau_c^*$ indicates the number of periods between the two parameters.

I can point out three main arguments regarding the largest differences on the methods mentioned above. Firstly, the method by Alkema et al (2011) represented by the Equation 2, accounts for the local and global maximum on the TFR above a TFR of 5.5 children per woman and when the condition is not reached, the model sets τ_c to be equal to the period 1950-1955. In this research I used data from the US Census Bureau and as I pointed out on the Section 3.2, the retrieved data sets of some countries carry out the same TFR value on consecutive periods over the time implying a lack of variation on fertility. In consequence, Equation 2 does not find a maximum, setting τ_c to be 1950-1955; this is the particular case of Niger and Uganda. A detailed calculation of every τ_c for all the projection countries is described in Appendix D

Secondly, since the method was originally developed to be used by the United Nations fertility projections, it assumes that the input data for the model is complete by including values starting on the period 1950-1955 with a total of 13 observed 5-year periods as of 2015 and that also accomplishes with the established data quality standards (United Nations, 2015). However, when using data with less 13 observations, Equation 2 cannot select a proper τ_c by implying that the fertility transition of the country has started in the period 1950-1955. For instance, the data used in this research contains an average of 8 observations and in some countries such as Cameroon, Mali and Sao Tome and Principe there is not a clear drop on the fertility within these few observations so in consequence, Equation 2 sets τ_c to be the period 1950-1955. In addition to this, the Equation 2, works the other way around in countries that exhibit even less observations, for instance, South Sudan counts with only two observations (2005-2010 and 2010-2015), so τ_c in this country is set to be the period 2005-2010, even though there is not enough information that supports the beginning of the fertility transition during that period. This finding implies that the reliability of selection of τ_c is based on the availability of the full time series containing 13 periods, which indeed contrasts with the spirit of the model due to the fact that the Bayesian Hierarchical model used in the simulation of the TFR trajectories, does not require much data and it can be improved and reinforced with additional information based on the expertise of the researcher, which is incorporated via the parameters of the decline function. Additionally, by setting τ_c as a fixed period based on the first observation of the United Nations' World Population Prospects data sets, the Equation 2 does not reflect the particular fertility behavior of the country; this is particularly noticed with Niger, Rwanda and Tanzania, in which the beginning of the fertility transition is broadly discussed and pointed out in many studies that fertility decline has stalled between 1995 and 2005 (Sneeringer, 2009).

Thirdly, Equation 2 chooses τ_c to be within the observed periods of the time series, however there are countries in which there is not supported that the fertility transition has already started since there is not a clear drop on fertility that indicates it. Setting the beginning of the fertility transition based only on the observation of fertility trends is not supported since the fertility behavior of a country is not an isolated demographic event but it is also consequence of other factors such as infant mortality as the Demographic Transition theory argues and many other studies I pointed out in the Section 2.1. In consequence, the selection of τ_c by the Equation 2 in countries like Niger and Nigeria is imprecise due to the fact that the method itself ignores the whole demographic context of those countries.

3.1.6. Software

The projections have been performed using the statistical software R and the most recent version of the package bayesTFR developed by Sevcikova (2015). This software incorporates the method developed by Alkema (2011) and Raftery (2013) and uses historical data to simulate a posterior distribution of total fertility rates for the selected countries simultaneously and allows the use of own data, to impute missing data or to apply the methods to sub-national datasets.

The R package projects the TFR in two different modules and the user can decide of using both modules or only one, depending on the characteristics of his research. The first module projects the fertility of the countries that are in the second stage of the fertility transition using the Bayesian Hierarchical model explained in this study, whereas the second module do so but for the countries that are in the third stage of the fertility transition. In this research only the first module is used since the scope of this research is forecasting the fertility in second stage countries. Moreover, the coding of the program was modified in order to incorporate the algorithm that calculates the starting period of the fertility decline ($\tau_c{}^*$) described in the section 3.1.4. A detailed description of the programming is provided in the Appendix B

3.2. Data

3.2.1. Required data

According to the selected countries mentioned in Chapter 2, the model will include information of 64 countries all around the world. The data has to show year by year the official demographic indicators of fertility and mortality prevalent in each country, hereof, time series of the Total Fertility Rate (TFR) and Infant Mortality (IMR) from 1960 until 2014 are required. In order to introduce the proper information to the model, the fertility time series have to be grouped in 5-year observations just as the world population projections of the United Nations (2013) are presented.

3.2.2. Data Collection

The data was retrieved from the US Census Bureau (USCB), which provides international demographic data collected from a variety of sources and evaluated, with particular attention to internal and temporal consistency, particularly, the estimates regarding mortality levels have been derived in a variety of ways, depending on data available on each county. For statistically more developed countries, data bases on mortality are taken from vital registration systems. For less statistically developed countries, the USCB develops mortality estimates using a combination of data sources such as nationally-representative household surveys, censuses and vital registration data. Additionally, in order to capture unusual increases in mortality, or demographic "shocks," special tallies and research reports from various national and international organizations are used (US Census Bureau, 2013).

Fertility estimates are, as in the case of mortality, dependent on the availability of data and on its level of detail. The USCB (2013) collects data from different sources, for instance, some countries have vital registration systems which produce data that is reliable, accurate, and which capture the timing of births with relative precision. Such data tend to be released on a regular, usually annually. In cases where births are not available, official government estimates of fertility, such as age-specific fertility rates (ASFRs) published in country statistical yearbooks or other regional statistical data bases are accepted and taken as it is. Fertility for statistically less developed countries is estimated using data from surveys, censuses and, to a lesser degree, vital registration systems.

Additionally, and in order to capture unusual changes in fertility levels and patterns, information from special surveys as well as epidemiological studies is taken into account. In general terms, the US Census Bureau assures the consistency, reliability and data quality of the retrieved data.

3.2.3. Description of the collected data

The intention of the research is to take 50 years of observations of the TFR and the IMR, however, due to the lack of available information, some countries present fewer observations, leading in an average of 30 observations of both demographic indicators per country, particularly, countries like Sudan and South Sudan have available data since 2008; other countries exhibit the same value of TFR in several years, for instance, Niger shows the same TFR from 1977 to 2006. Notwithstanding this findings, the infant mortality time series show more consistency along the time. A description and analysis of the fertility rates is shown in the following paragraphs.

3.2.3.1. Low fertility countries

The countries labeled as "low fertility countries" in the Table 1 are the ones that are in the third stage of the fertility transition, and they have presented a TFR between 1.2 and 2.0 children per women in the most recent period. Most of the countries included in this category have shown a maximum fertility decline within a five-year of observations below 0.5 children per women. Particularly, Iran is an outlier that have presented a drop on fertility greater that one child per women; this country has been included in this category due to the fact that most recent TFR observed in 2014 is around or below the replacement level of 2.1 children per women and in that sense they are considered as low fertility countries.

Table 5. Descriptive statistics of the low fertility countries

Country	First TFR registered	First year registered	Maximum decrease on a 5-year period*	5-year period of the maximum decrease **	Current TFR	Median
Armenia	2.49	1989	-0.39	1990-1995	1.64	1.75
Aruba	1.94	1981	-0.28	1995-2000	1.84	1.96
Azerbaijan	3.11	1989	-0.81	1990-1995	1.91	2.04
Bahamas, The	2.80	1980	-0.37	1980-1985	1.97	2.22
Barbados	2.02	1980	-0.25	1980-1985	1.68	1.68
Brazil	5.06	1970	-0.68	1980-1985	1.79	2.68
Chile	2.77	1982	-0.30	1995-2000	1.84	2.22
Georgia	2.23	1989	-0.34	1990-1995	1.77	1.78
Iran	6.57	1986	-1.93	1990-1995	1.85	2.34
Korea, South	1.59	1990	-0.28	1995-2000	1.25	1.27
Lebanon	5.54	1970	-0.73	1970-1975	1.74	2.69
Saint Lucia	5.16	1980	-0.91	1980-1985	1.77	2.61
Thailand	2.05	1990	-0.26	1995-2000	1.50	1.57
Trinidad and Tobago	3.17	1980	-0.69	1985-1990	1.71	1.82
Vietnam	3.79	1989	-0.76	1990-1995	1.85	2.27

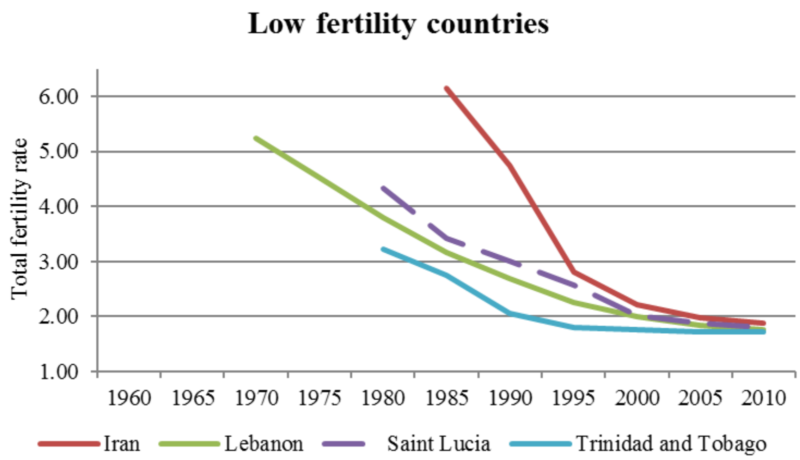
Source: Self- computed with information retrieved from the US Census Bureau (2014)

*The mean of each 5-year period was first calculated, then the fertility decline is given by $f_{t+1} - f_t$ and it is measured in terms of children per women. Therefore, the maximum decrease on the observed means was picked in each country.

**The 5-year period in which the maximum decrease was observed.

The graph 6 presents the path that fertility followed in some of the selected low-fertility countries; we can see the convergence to levels around 2.0 children per women. It is important to remark the drop in fertility that Iran showed, halving the TFR from level of 6.00 to 3.00 children per women in less than ten years.

Graph 6. Total fertility rate of some low-fertility countries in the period 1960-2014

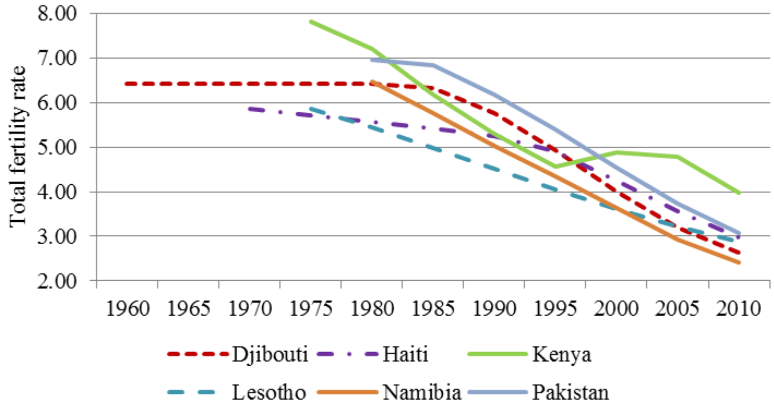


Source: Self- computed with information retrieved from the US Census Bureau (2014).

3.2.3.2. Transition countries

The median TFR of the selected transition countries was between 2.0 and 5.0 children per woman along the time series. The average maximum decrease observed in a five-year period is 0.64 children per women, which indeed indicates that the fertility transition has started in this set of countries. It is important to remark that Pakistan and Swaziland show similar fertility trajectories pointing out that there are many analogous patterns among different world regions.

Graph 7. Total fertility rate of some transition countries in the period 1960-2014



Source: Self- computed with information retrieved from the US Census Bureau (2014)

Table 6. Descriptive statistics of the transition countries

Country	First TFR registered	First year registered	Maximum decrease on a 5-year period*	5-year period of the maximum decrease **	Current TFR	Median
Bolivia	4.77	1992	-0.46	2000-2005	2.80	3.74
Central African Republic	5.88	1975	-0.29	2000-2005	4.46	5.43
Comoros	7.02	1980	-0.83	2005-2010	3.76	5.25
Curacao	2.45	1992	-0.34	1995-2000	2.08	2.23
Djibouti	6.41	1960	-0.94	1995-2000	2.47	6.37
Gabon	4.14	1969	-0.44	1985-1990	4.49	4.91
Gambia, The	6.82	1983	-0.64	2005-2010	3.85	5.76
Ghana	6.93	1970	-0.79	1985-1990	4.09	5.23
Guatemala	5.91	1981	-0.66	1995-2000	2.99	4.87
Guinea-Bissau	5.85	1979	-0.35	2005-2010	4.30	5.49
Haiti	5.89	1971	-0.69	2000-2005	2.79	5.25
Honduras	7.02	1974	-0.61	2000-2005	2.86	4.94
Jordan	4.70	1994	-0.35	1995-2000	3.16	3.88
Kenya	7.80	1979	-1.02	1980-1985	3.54	4.98
Lesotho	6.00	1976	-0.47	1985-1990	2.78	4.23
Madagascar	6.94	1975	-0.61	1995-2000	4.28	6.10
Mauritania	6.48	1977	-0.38	2000-2005	4.07	5.46
Namibia	6.64	1980	-0.72	1985-1990	2.25	4.35
Pakistan	6.96	1981	-0.85	1995-2000	2.86	5.33
Philippines	5.09	1980	-0.50	1980-1985	3.13	3.87
Sudan	4.69	2008	-0.45	2005-2010	3.92	4.30
Swaziland	6.89	1976	-0.69	1985-1990	2.88	4.76
Tajikistan	4.31	1989	-0.80	1990-1995	2.76	3.36
United Arab Emirates	5.48	1986	-1.27	1985-1990	2.36	2.57
Zimbabwe	6.56	1982	-1.13	1985-1990	3.56	3.98

Source: Self- computed with information retrieved from the US Census Bureau (2014)
*The mean of each 5-year period was first calculated, then the fertility decline is given by $f_{t+1} - f_t$ and it is measured in terms of children per women. Therefore, the maximum decrease on the observed means was picked in each country.
**The 5-year period in which the maximum decrease was observed.

The slope of the decline in the transition countries is clearly steep; particularly Namibia, Dijbouti and Pakistan have shown a similar uniform decline during the last 30 years, contrary to Kenya that have presented a mix trend with a mild rise during the 90s followed by a decline in the following years.

3.2.3.3. Projection countries

These countries are still showing high TFR levels in recent years and they have not shown a significant decline along the time; Liberia and Rwanda and Somalia have exhibited a decrease above one child per woman in 5-year that could indicate the onset of the transition phase in these three countries. In spite of this finding, I have included these countries in the projection for two reasons, firstly, because they are part of the Sub-Saharan region which is the aim of study of this research and secondly, intuitively, they will introduce volatility to the parameters associated to the decline function associated with the statistical model. A comparison of the annual maximum decrease among all the countries is presented in the Graph 8, in which we could observe the set of countries with a decrease above the 0.5 child per women in the TFR, these observations are useful to calibrate the parameter of initial decrease U_c in the statistical model and to set the start year in which the fertility decreased. It is important to remark that these results are in line with the results presented in the table 3 of the section 3.1.4, in which a detail description of the calculation is provided.

Table 7. Descriptive statistics of the projection countries

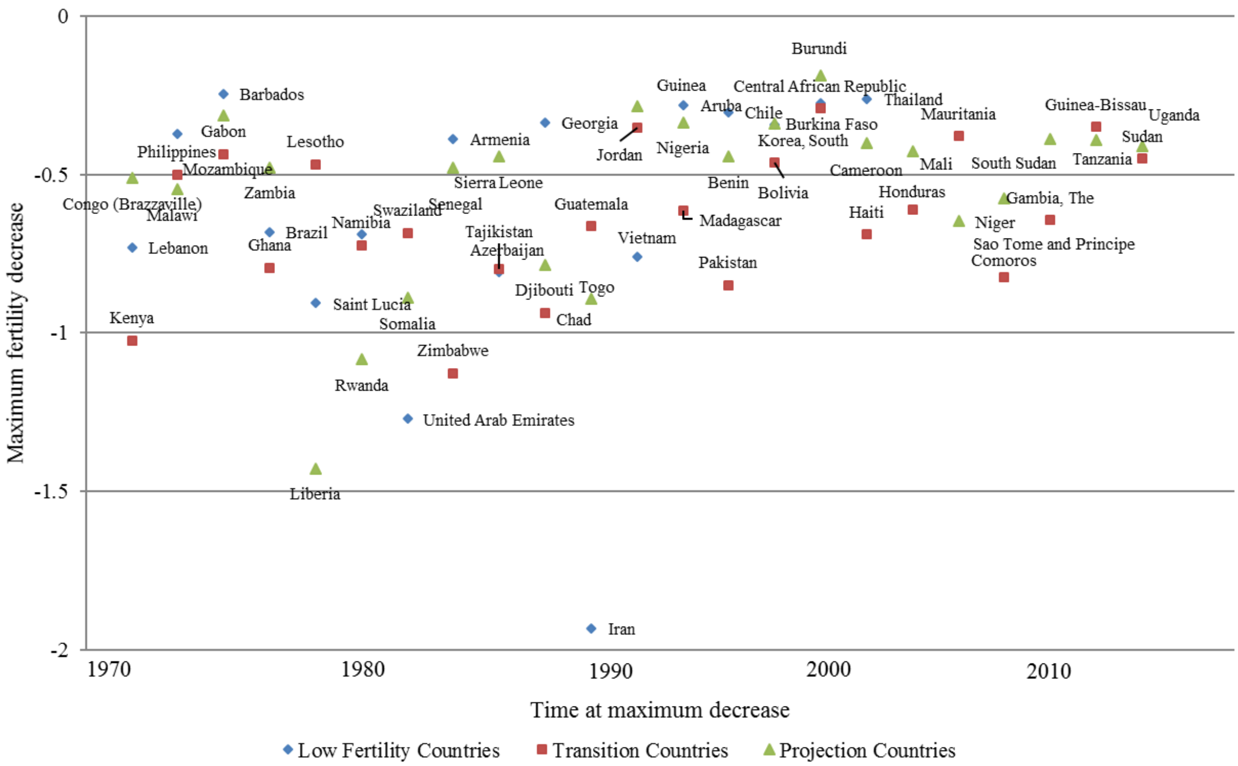
Country	First TFR registered	First year registered	Maximum decrease on a 5-year period*	5-year period of the maximum decrease **	Current TFR	Median
Benin	7.08	1979	-0.44	2005-2010	5.04	6.40
Burkina Faso	7.76	1976	-0.34	2005-2010	5.93	7.13
Burundi	6.95	1979	-0.19	2005-2010	6.14	6.62
Cameroon	6.38	1976	-0.40	2005-2010	4.82	5.76
Chad	6.16	1964	-0.89	2000-2005	4.68	6.51
Congo (Brazzaville)	6.29	1974	-0.51	1975-1980	4.73	5.06
Guinea	6.05	1960	-0.28	2000-2005	4.93	5.91
Liberia	6.38	1974	-1.43	1985-1990	4.81	6.38
Malawi	7.72	1977	-0.55	1980-1985	5.66	6.54
Mali	7.64	1977	-0.43	2005-2010	6.16	7.39
Mozambique	6.49	1980	-0.31	1980-1985	5.27	5.64
Niger	7.97	1977	-0.65	2005-2010	6.89	7.97
Nigeria	7.20	1960	-0.34	2000-2005	5.25	6.66
Rwanda	8.07	1978	-1.08	1985-1990	4.62	5.96
Sao Tome and Principe	6.35	1981	-0.57	2005-2010	4.67	6.16
Senegal	7.29	1976	-0.48	1990-1995	4.52	6.07
Sierra Leone	6.50	1974	-0.44	1990-1995	4.83	6.02
Somalia	7.25	1975	-0.89	1985-1990	6.08	7.08
South Sudan	6.10	2008	-0.39	2005-2010	5.43	5.77
Tanzania	6.90	1978	-0.39	2005-2010	4.95	5.80
Togo	7.40	1981	-0.79	1990-1995	4.53	5.41
Uganda	7.55	1969	-0.41	2005-2010	5.97	7.33
Zambia	7.20	1980	-0.48	1980-1985	5.76	6.04

Source: Self- computed with information retrieved from the US Census Bureau (2014)

*The mean of each 5-year period was first calculated, then the fertility decline is given by $f_{t+1} - f_t$ and it is measured in terms of children per women. Therefore, the maximum decrease on the observed means was picked in each country.

**The 5-year period in which the maximum decrease was observed.

Graph 8. Maximum fertility decrease among the selected countries



Source: Self- computed with information retrieved from the US Census Bureau (2014)
The fertility decline was calculated by $f_{t+1} - f_t$ and it is measured in terms of children per women.

4. Results

In the section 4.1 I introduce the results of the projections made with the model that incorporates the mortality trends in the forecast. The results are presented for the total Sub-Saharan region and for some selected countries with the aim of show the general fertility path the region will follow as well as an analysis of the model implemented. The complete fertility trajectories of the remaining countries are stated in the Appendix C.

In the section 4.2 I performed a comparison between the original model presented by Alkema (2011) and the one proposed in this research, which incorporates the mortality in order to identify the differences and apprise for possible improvements. Additionally, the figures are compared with the United Nations' World Population Prospects 2015.

4.1. Sub-Saharan Africa

The fertility within African Sub-Saharan shows a general fall within the next 40 years, passing from a median TFR of 5.09 children per woman in 2015 to 4.23 in 2025 and ending with a TFR of 3.35 children per women in the year 2050, exhibiting a constant annual decrease of 0.2 children per women. The distribution of the decline function for the region indicates that during 2050, the TFR should lie between the interval of 1.89 and 4.55 children per woman with a probability of 0.95, implying that as the forecast goes further in time, the projection interval goes broader, accounting for more uncertainty in long-term projections.

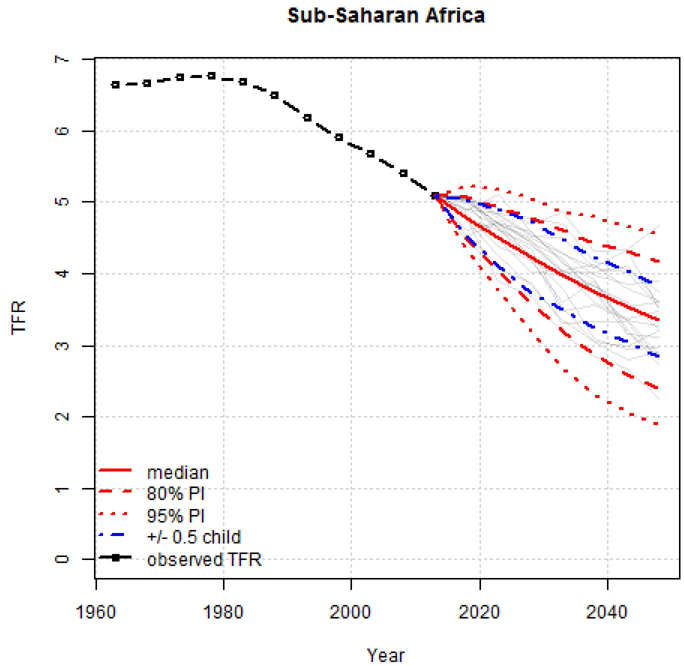
Furthermore, stressing the projections by adding and subtracting half children is a common practice made by United Nations. This was also performed on the projections presented in this research and by adding 0.5 children per woman the TFR will be 4.73 and 3.85 children per woman during 2025 and 2050 respectively. Analogously, subtracting 0.5 children per women to the projected TFR will lead to levels of 3.73 during 2025 and 2.85 during 2050 children per woman.

Table 8. Fertility projections of Sub-Saharan Africa (2010-2050)

Country/Period of projection	2010-2015	2015-2020	2020-2025	2025-2030	2030-2035	2035-2040	2040-2045	2045-2050
Benin	5.22	4.83	4.48	4.17	3.88	3.60	3.36	3.14
Burkina Faso	6.07	5.67	5.29	4.91	4.55	4.21	3.90	3.61
Burundi	6.23	6.02	5.78	5.51	5.21	4.87	4.53	4.18
Cameroon	4.94	4.62	4.31	4.02	3.76	3.54	3.31	3.13
Chad	4.93	4.28	3.73	3.29	2.97	2.71	2.50	2.34
Congo	4.82	4.54	4.29	4.07	3.85	3.65	3.47	3.29
Guinea	5.04	4.68	4.30	3.92	3.60	3.29	3.00	2.78
Liberia	5.02	4.60	4.19	3.85	3.54	3.26	3.02	2.84
Malawi	5.77	5.44	5.14	4.88	4.62	4.38	4.17	3.98
Mali	6.35	5.94	5.54	5.14	4.79	4.46	4.16	3.88
Mozambique	5.40	5.14	4.86	4.61	4.35	4.09	3.86	3.64
Niger	7.16	6.50	5.87	5.29	4.76	4.31	3.91	3.55
Nigeria	5.38	5.08	4.83	4.58	4.35	4.10	3.91	3.70
Rwanda	4.81	4.39	4.06	3.76	3.48	3.27	3.06	2.89
Sao Tome and Principe	4.94	4.46	3.99	3.58	3.22	2.95	2.72	2.54
Senegal	4.69	4.34	4.03	3.75	3.51	3.30	3.11	2.94
Sierra Leone	4.90	4.56	4.24	3.95	3.69	3.46	3.26	3.05
Somalia	6.26	5.80	5.35	4.92	4.53	4.17	3.84	3.55
Sudan	4.17	3.84	3.55	3.26	3.04	2.84	2.67	2.55
United republic of tanzania	5.08	4.77	4.51	4.26	4.03	3.79	3.58	3.41
Togo	4.64	4.23	3.87	3.58	3.32	3.10	2.91	2.75
Uganda	6.14	5.69	5.26	4.84	4.45	4.10	3.77	3.48
Zambia	5.85	5.56	5.31	5.05	4.82	4.60	4.40	4.22

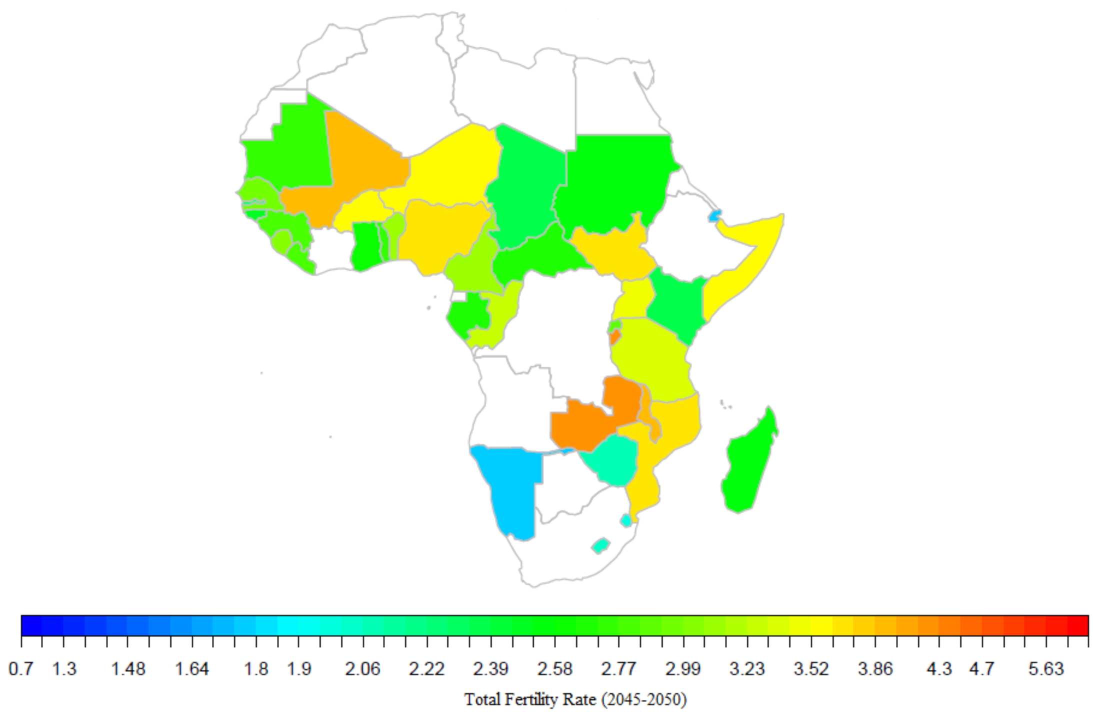
The median of each probability distribution was taken and shown in this table

Graph 9. Fertility trajectories of Sub-Saharan Africa 1960-2015



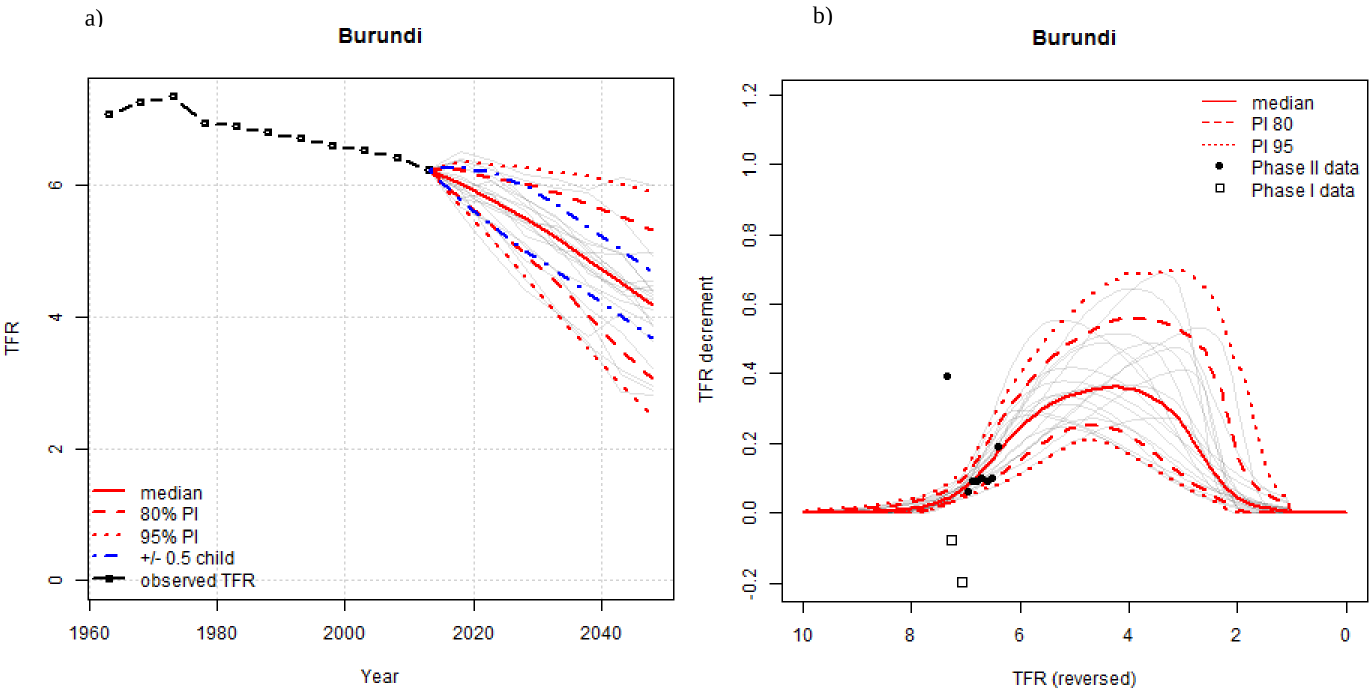
In spite of the constant pace of the fertility decline, there are some countries such as Burundi and Zambia that will still showing a median TFR above to 4.0 children per woman in 2050 and as shown in the Graph 9, most of the countries show fertility levels above 3.0 children per women. Countries like Cameroon, Sao Tome and Principe and Sudan exhibit fertility levels around 2.5 children per women. In the following sub-sections, I analyzed in detail the fertility path of some Sub-Saharan countries.

Graph 10. Projected fertility levels on Sub-Saharan Africa for the period 2045-2050



4.1.1. Burundi

Graph 11. Fertility projections of Burundi (2015-2050)

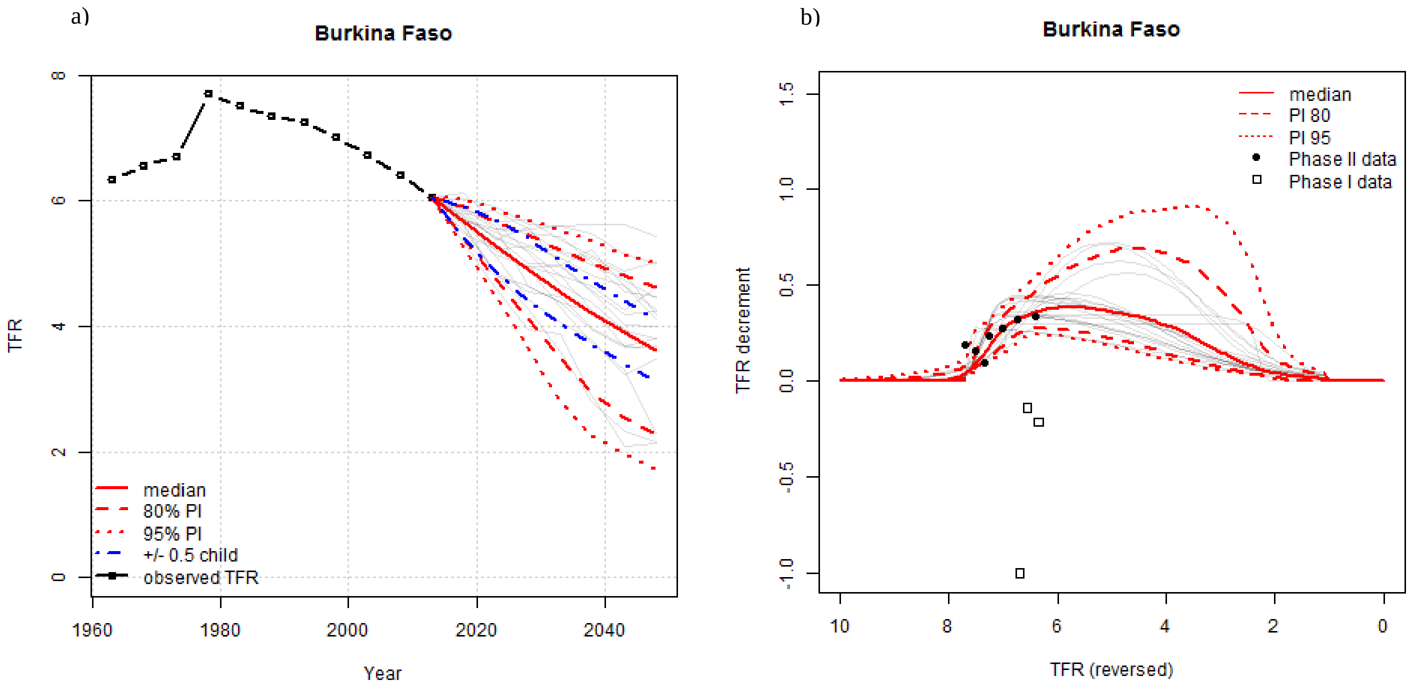


Since the decade of 1960, Burundi had showed high fertility levels above the 6.0 children per women with a gradual decrease over the time and since the period 1990-1995 the pace of the decline speeded up and the slope of the median TFR becomes steeper in the projected years passing from 6.23 children per woman in 2015 to 5.51 in 2025 and 4.22 in 2050 and during this last year it is expected that the TFR of Burundi fluctuates between 2.52 and 5.81 children per woman with 0.95 probability. Furthermore, stressing the fertility trajectory with a half a child more, the TFR will reach levels of 4.71 during 2050.

The Graph 11b indicates a slow decrease on fertility from levels above 6.0 children per woman reaching a maximum decrement on fertility of 0.3 children per woman observed at a median TFR around 4.7 children per women and then the speed of the decrease went down converging to levels close to 2.0 children per woman.

4.1.2. Burkina Faso

Graph 12. Fertility projections of Burkina Faso (2015-2050)

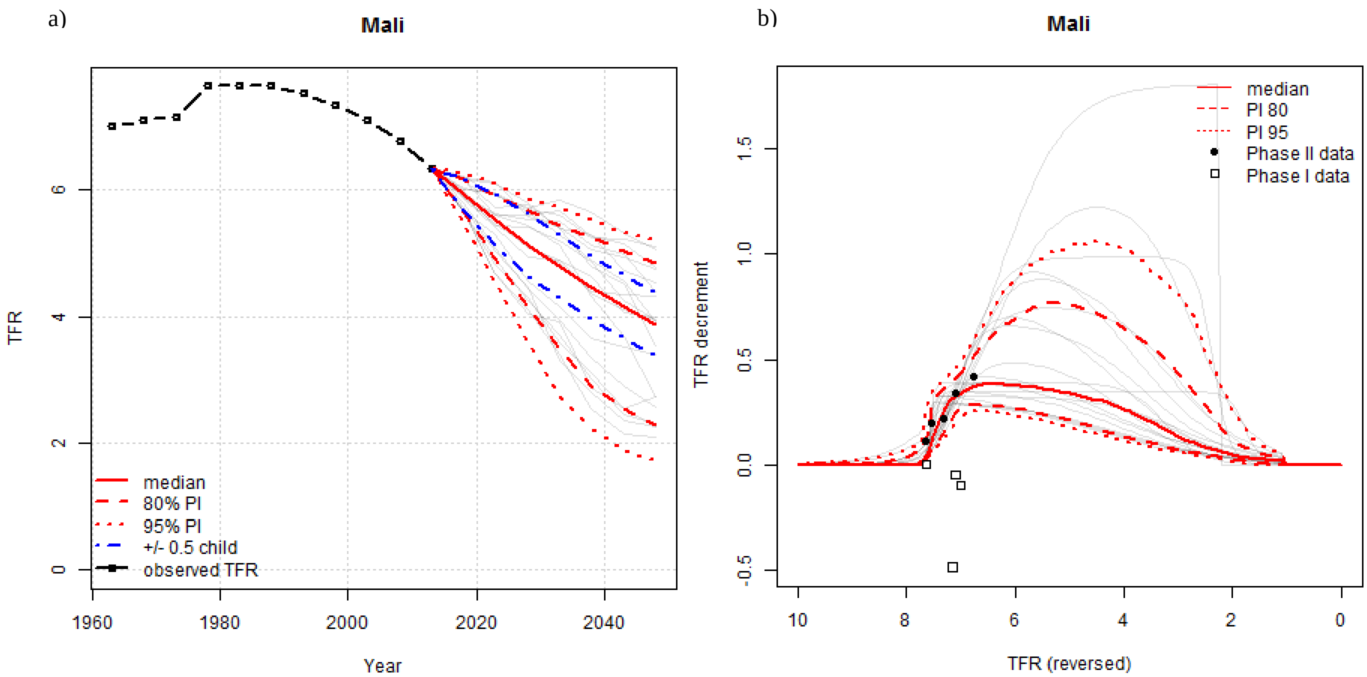


The fertility transition in Burkina Faso has pointed out to start during the period 1985-1990 with levels up to 7.7 children per woman and since that year the decline has followed an uniform decrease. The projection made in this research states a fertility trajectory that pass from a TFR of 6.07 in 2015 to 3.61 children per woman in the period 2045-2050, implying that during that period of time the fertility in the country will be between 1.73 and 5.01 children per woman with 0.95 of probability. The fertility decline in Burkina Faso has been pointed out in some studies (Sneeringer, 2009; Lesthaeghe, 2014) to be in the transition phase, which is in line with the projections performed in this research.

The speed of the fertility decline illustrated in the Graph 12b, indicates that Burkina Faso has started with a decline around of a median TFR of 7.0 children per woman and rapidly reached the maximum drop on fertility of 0.3 followed by slowly lessening in lower levels of TFR.

4.1.3. Mali

Graph 13. Fertility projections of Mali (2015-2050)



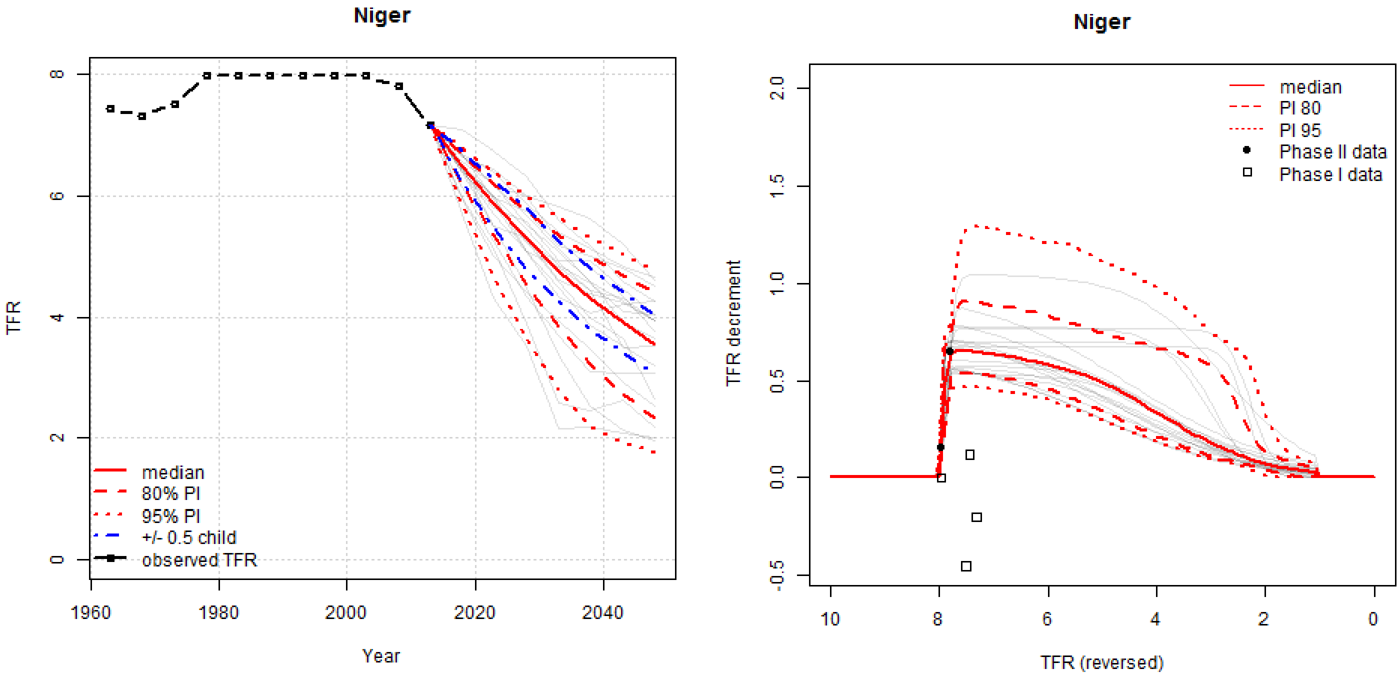
The method to select the parameter $\tau_c{}^*$ introduced I this research establish the onset of the fertility transition in Mali within the period 1990-1995 and this could be visually identified and supported in the graph 13a. After this period, TFR has suffered an average decrease of 0.35 children per 5-year period projected, moreover, the model project a median TFR of 5.14 children per woman in 2025 and 3.88 in 2050. The TFR in that year will be between 1.872 and 5.20 children per woman with a 0.95 probability.

The Graph 13b captures the pace of the fertility decrease on Mali, reaching the highest drop on the median TFR at levels of 7.0 children per woman followed by a slow decline during subsequent fertility levels. It is important to point out that the decline function in this country presents a wide variation on TFR levels from 6.0 to 3.0 due to the uncertainty that the future fertility path will follows.

4.1.4. Niger

Graph 14. Fertility projections of Niger (2015-2050)

a) b)



Niger is one of the countries of the Sub-Saharan Africa region that accounts for high levels of uncertainty regarding its future fertility path; and this is due to the fact that since during three decades, the TFR remained constant. As I pointed out in the Section 3.1.5, in this specific country the value of $\tau_c{}^*$ estimated with the method introduced in this study differs considerably to the one stated in Alkema et al (2011) that fails due to the lack of a clear drop on fertility all over the time.

By incorporating the decrease on mortality, $\tau_c{}^*$ the fertility decline in Niger has been identified to start during the period 2000-2005, which also can be supported by observing the trend on the graph 14a. Since this point, the slope curve becomes steeper, passing from a median TFR of 7.16 during the period 2010-2015 to a TFR of 5.30 during 2025-2030 and 3.55 children per women in 2045-2050 and during that last period is projected to be between 1.82 and 4.82 children per woman with a probability of 0.95.

Additionally, it can be observed in the graph 14b that the decline function portrays an abrupt drop of 0.5 children per woman on a TFR levels up to 8.0, representing a breaking point on the fertility path of the country. Moreover, the projection interval of the TFR in Niger is particularly large, since the median TFR could goes up or down by 1 child per woman with a probability of 0.95 implying a high degree of uncertainty along the whole time spam. A further discussion on the Niger's fertility trends is given on the section 4.2.

4.2. Comparison with the World Population Projections and improvements

One of the main purposes of this research is to prove that the incorporation of infant mortality trends implies a better fit on projecting the future fertility trajectories on the sub-Saharan African Region. In this regard, and in order to measure the impact of incorporating new information to the fertility trajectory, I ran two different model specifications; the original model (identified as the "First Approach model" later on) introduced by Alkema (2011) and replicated in the R package "bayesTFR" (Sevcikova, 2015). The second model (referred as "Mortality" model), the parameter τ_c^* was modified by setting up different starting periods on fertility declines in response to infant mortality trends as I pointed out in the section 3.2 (for more details on the code see Appendix B). Additionally, I brought to the comparison the fertility figures stated in the United Nations' World Population Prospects 2015 in a way to point out a sensitivity analysis.

In comparing the medians of the Mortality and the First Approach models I encountered that in the period 2015-2020 the Mortality TFR is in average one per-cent higher than the TFR of the First Approach model, and as the projection goes further, the gap between two models goes up ending with a difference on 7% between the two models in the period 2045-2050, being the Mortality TFR higher than the First Approach model. Particularly, in table 9 we can appreciate that the median TFRs of Burkina Faso, Burundi, Mali and Nigeria gotten with the Mortality model are in all cases higher than the First Approach model, furthermore the 95% projection interval on the Mortality model is narrower than the First Approach model indicating a better convergence of the figures diminishing the level of uncertainty on the projections. These observations let us see that there is a postponement on the fertility decline due to the incorporation of the mortality trends and in the Mortality model, the pace of the decline is slowed down.

Table 9. Comparison of the outcomes of the projection of the different models for the periods 2025-2030 and 2045-2050

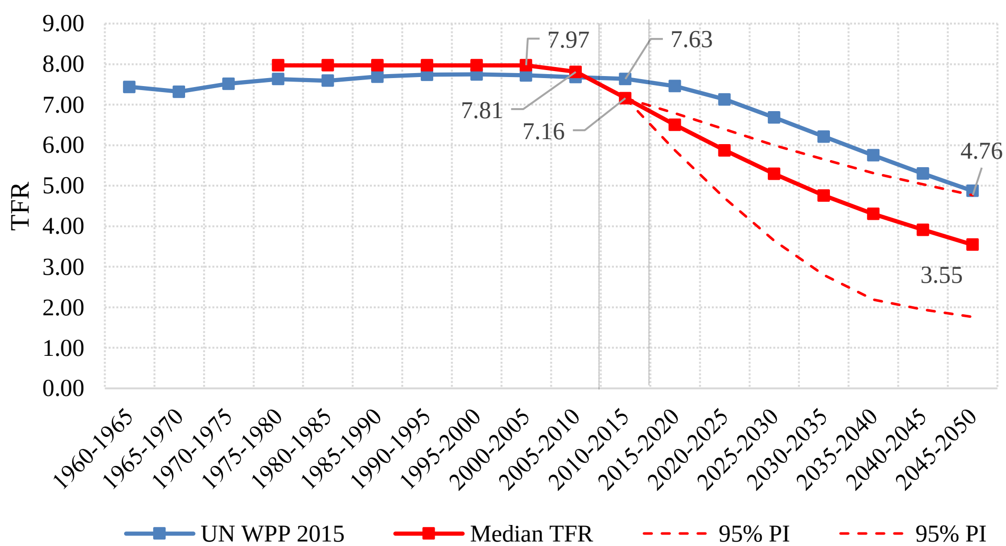
	2025-2030						
	United Nations	First Approach			Mortality		
	Median	Median	Lower	Upper	Median	Lower	Upper
Burkina Faso	**4.48**	**4.84**	*3.63*	*5.80*	**4.91**	*3.66*	*5.69*
Burundi	**4.89**	**5.34**	*4.30*	*6.34*	**5.51**	*4.62*	*6.25*
Mali	**5.03**	**5.07**	*3.63*	*5.98*	**5.14**	*3.62*	*5.86*
Niger	**6.68**	**5.22**	*3.46*	*6.13*	**5.29**	*3.64*	*6.00*
	2045-2050						
	Median	Median	Lower	Upper	Median	Lower	Upper
Burkina Faso	**3.34**	**3.46**	*1.81*	*5.01*	**3.61**	*1.74*	*5.01*
Burundi	**3.69**	**3.80**	*2.05*	*5.92*	**4.18**	*2.50*	*5.90*
Mali	**3.57**	**3.68**	*1.76*	*5.17*	**3.88**	*1.72*	*5.20*
Niger	**4.87**	**3.40**	*1.77*	*4.86*	**3.55**	*1.76*	*4.76*

The United Nations figures were retrieved from the Medium variant of the World Population Prospects, 2015.
The columns indicated as the Lower and Upper category represent the 0.95 lower and upper boundary of the projection interval.

Additionally, by comparing the Mortality model with the United Nations medium variants, we can point out that in general terms, the median TFR of the Mortality model is higher than the United Nations figures, however there are cases in which this statement is not true. For instance, the trend in Niger is the other way around since The United Nations has estimated a TFR that is above the median TFR of the Mortality model and it is even higher than the 95 per-cent upper bound of the probability distribution.

To understand the particular case of Niger, we can distinguish two parts of TFR trajectory portrayed in the Graph 15. Firstly, within the periods going from 1975-1980 to 2010-2015, the red line shows the observed TFRs retrieved from the US Bureau Census and secondly, the line is continued with the values gotten in the Mortality model. In this regard, we can observe that until the period 2005-2010 the red line is higher than the UN estimates (blue line) and since this point the trend goes the other way around passing from a TFR of 7.81 in 2005-2010 to 7.16 in 2010-2015 children per woman indicating a drop on the fertility by 0.6 children per woman in a 5-year period. The selected $\tau_c{}^*$ for Niger in the Mortality model is pointed to be in the period 2000-2005 and since this period the model takes into account the observed drop and continues with the decline making the slope of the TFR trajectory steeper and speeding up the fertility transition ending in levels of a median TFR of 3.55 children per woman, hence the 95% projection interval follows the trend given by the median.

Graph 15. Comparison between the fertility trajectories in Niger portrayed in the Mortality model and the United Nations' World Population Prospects (1960-2050)



In contrast, the UN estimates indicates a TFR of 7.63 children per woman on the period 2010-2015, maintaining the same fertility levels and only showing a slow decrease on the following projected periods ending with a TFR of 4.76 children per woman in the period 2045-2050. To sum up, I can point out that the main difference between the Mortality model and the United Nations figures lays on the observed drop portrayed in the US Bureau Census fertility time series since the decline function of the mortality model takes into account this trend along with the information of other similar countries in order to make the projections.

5. Conclusion

There are two main findings on this research, the first one is regarding to the general fertility path on Sub-Saharan Africa, which indeed exhibits a declining trajectory along the next 40 years. The projection shows that as at 2025, the whole region will show an average median TFR around 4.3 children per woman and during 2050, the region will exhibit a median TFR of 3.23. However, in countries like Burundi and Zambia, the pace of the fertility transition is moderate and gradually slowed down ending with levels up to 4 children per woman during 2050.

The second finding is the actual improvement that the fertility projections suffer by taking into account the observed regularities on the drop on the child mortality. In this regard, I have to do some remarks, for instance that fertility data on the Sub-Saharan African region is scarce since there are no many sources and the available data is incomplete, particularly the data sets on fertility retrieved from the US Census Bureau and used in this study do not account for a good level of data quality as I mentioned on the section 3.2.3. The model used in this research (Alkema et al, 2011) use a deterministic rule to select the onset of the fertility transition, however, without enough observations this rule fails and the selection turns arbitrary implying an early decrease that in some cases has started 50 years ago. By incorporating infant mortality trends on the procedure, the onset of the fertility transition is better supported based on the full demographic context of the country by interrelating the demographic variables and making them dependent. This is particularly useful in countries like Niger and Burundi, that have not shown a drop on mortality during the observed periods and in which the level of uncertainty is high. The rule proposed in this research makes that the fertility drop is postponed and the pace of convergence to replacement levels is slowed down.

In this research, I proposed a way to describe the fertility transition in some Sub-Saharan countries by integrating child mortality patterns on fertility projections using a Bayesian model that also incorporates uncertainty to the projection. The aim of using this model was also to introduce a different view of a traditional issue, passing from the statistical perspective based in frequentist methods to the Bayesian approach by supporting the analysis in probabilities and taking advantage of the recent development on statistics. This was made by keeping in mind that human behavior and particularly fertility are random processes and their measurement implies uncertainty

Appendix A: Glossary of terms

- **Prior distribution:** Probability distribution that would express one's beliefs about this quantity before some evidence is taken into account.
- **Posterior distribution:** Probability distribution of an unknown quantity, treated as a random variable, conditional on the evidence obtained from an experiment or survey.
- **Simulation:** Is a way to model random events, such that simulated outcomes closely match real-world outcomes.
- **Markov chain Monte Carlo:** Algorithm for sampling from a probability distribution based on constructing a Markov chain that has the desired distribution.
- **Bayesian Hierarchical model:** Statistical model written in multiple levels (hierarchical form) that estimates the parameters of the posterior distribution using the Bayesian method.
- **Total fertility rate:** The total fertility rate uses the synthetic cohort approach and approximates knowing how many children women have had when they are all through which childbearing by using the age specific fertility rates at a particular date to project what could happen in the future if all women went through their lives bearing children at the same rate that women of different agers were at that date.
- **Mortality:** The extent to which people are unable to live to their biological maximum age.
- **Crude death rate:** Total number of deaths in a year divided by the average total population.
- **Infant mortality rate**: Number of deaths less than one year of age per 1,000 live births.

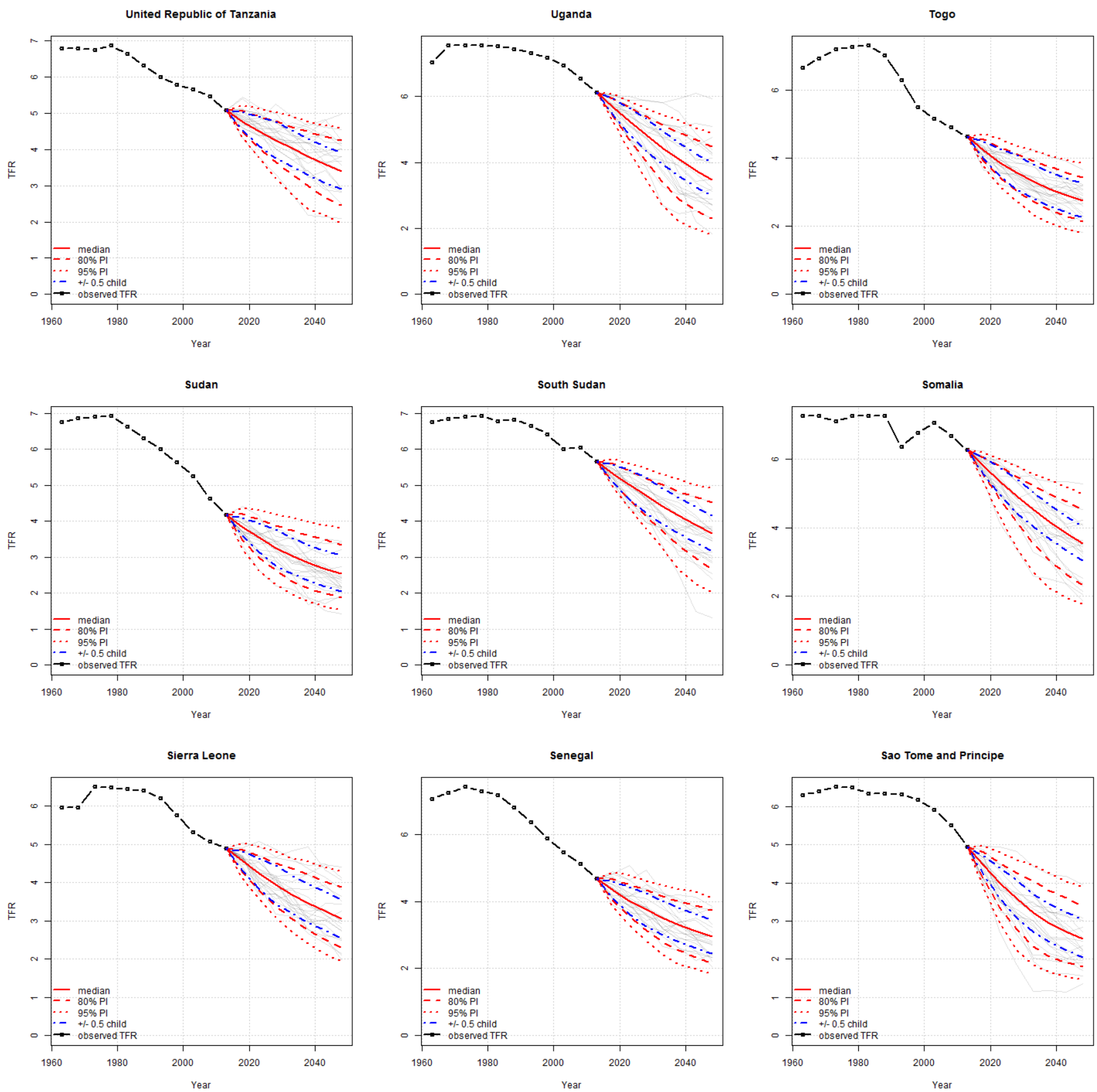
Appendix B: R code

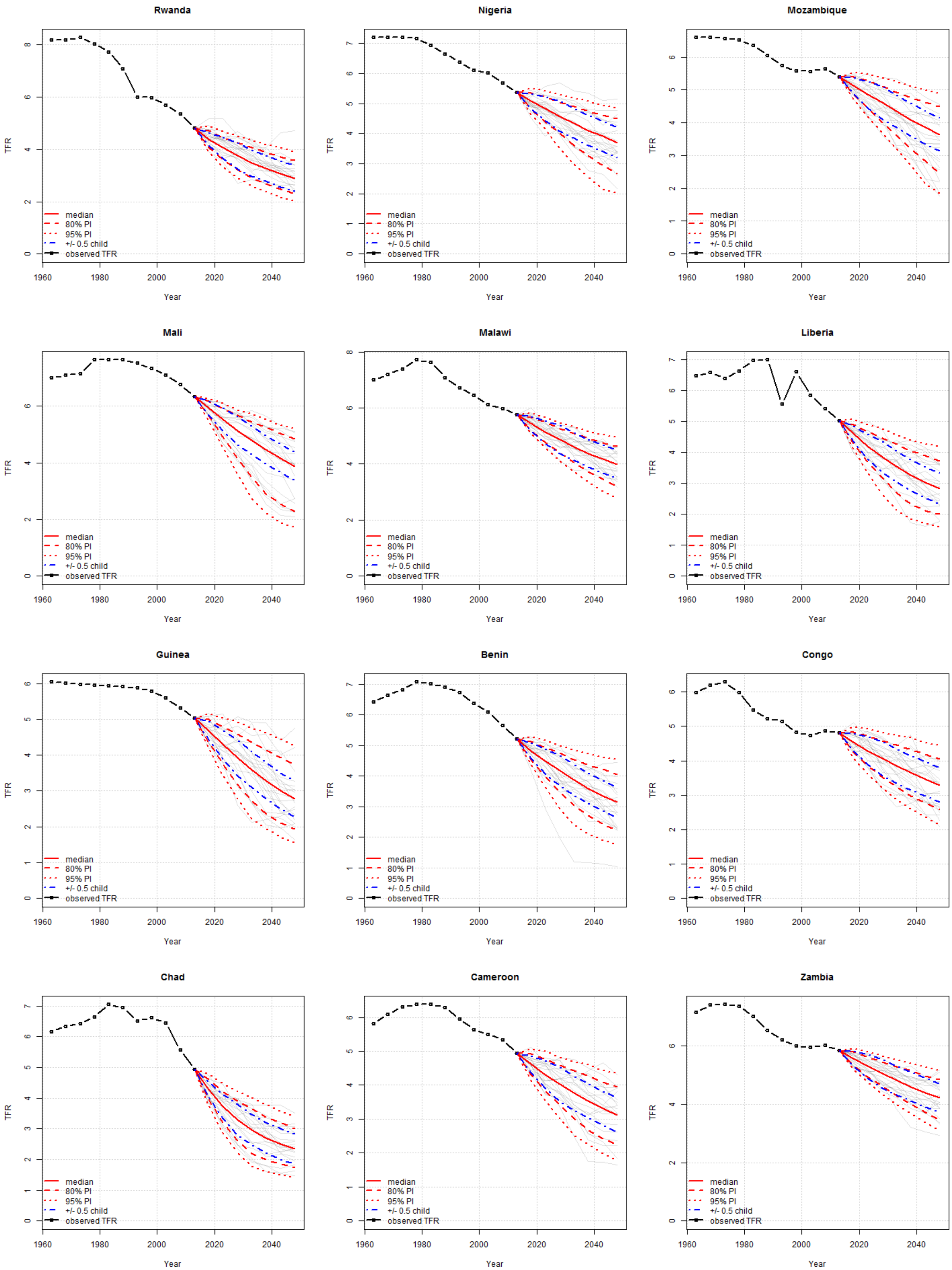
In order to calculate $\boldsymbol{\tau_c}^*$ according to the methodology introduced in the section 3.1.4, I modified the source code of the module mcmc_ini.R of the "bayesTFR" R package by adding in the line 194, the following code:

```
#########################################################Set of different tau_c for the projected countries

tau_c('Benin')=1985
tau_c('Burkina Faso')=1985
tau_c('Burundi')=1990
tau_c('Cameroon')=1985
tau_c('Chad')=1990
tau_c('Congo')=1985
tau_c('Guinea')=1980
tau_c('Liberia')=1990
tau_c('Malawi')=1990
tau_c('Mali')=1990
tau_c('Mozambique')=1990
tau_c('Niger')=2000
tau_c('Nigeria')=1990
tau_c('Rwanda')=1990
tau_c('Sao Tome and Principe')=1990
tau_c('Senegal')=1985
tau_c('Sierra Leone')=1995
tau_c('Somalia')=1990
tau_c('Sudan')=1995
tau_c('United Republic of Tanzania')=1990
tau_c('Togo')=1985
tau_c('Uganda')=1985
tau_c('Zambia')=1990

##################################################################################################
```

After the inclusion of these lines, I compiled the package, installed it and run it as the normal procedures making five Monte-Carlo Markov Chains with 10,000 simulations each.

Appendix C: Fertility trajectories of Sub-Saharan African countries




Rwanda
Nigeria
Mozambique
Mali
Malawi
Liberia
Guinea
Benin
Congo
Chad
Cameroon
Zambia
TFR
Year
median
80% PI
95% PI
+/- 0.5 child
observed TFR

Appendix D: Calculation of parameter τ_c according to Alkema et al. (2011)

Country / Year	1960-1965	1965-1970	1970-1975	1975-1980	1980-1985	1985-1990	1990-1995	1995-2000	2000-2005	2005-2010	2010-2015	M_c	$L_{c.t}$		τ_c
Benin	N/A	N/A	N/A	7.08	7.02	6.91	6.74	6.38	6.10	5.66	5.22	1975-1980	1975-1980		1975-1980
Burkina Faso	N/A	N/A	N/A	7.70	7.51	7.35	7.25	7.01	6.73	6.41	6.07	1975-1980	1975-1980		1975-1980
Burundi	N/A	N/A	N/A	6.95	6.89	6.80	6.71	6.61	6.52	6.42	6.23	1975-1980	1975-1980		1975-1980
Cameroon	N/A	N/A	N/A	6.38	6.38	6.29	5.95	5.64	5.50	5.34	4.94	1975-1980	Not found		1950-1955
Chad	6.16	6.34	6.42	6.65	7.05	6.94	6.51	6.62	6.45	5.56	4.93	1980-1985	1980-1985	1995-2000	1950 - 1955
Congo (Brazzaville)	N/A	N/A	6.29	5.99	5.47	5.23	5.15	4.83	4.74	4.87	4.82	1970-1975	1970-1975	2005-2010	1970-1975
Guinea	6.04	6.01	5.98	5.96	5.93	5.91	5.87	5.79	5.59	5.31	5.04	1960-1965	1960-1965		1960-1965
Liberia	N/A	N/A	6.38	6.64	6.97	6.99	5.56	6.60	5.84	5.42	5.02	1985-1990	1985-1990	1995-2000	1995-2000
Malawi	N/A	N/A	N/A	7.72	7.64	7.09	6.73	6.46	6.13	5.99	5.77	1975-1980	1975-1980		1975-1980
Mali	N/A	N/A	N/A	7.64	7.64	7.64	7.53	7.33	7.11	6.77	6.35	1975-1980	Not found		1950 - 1955
Mozambique	N/A	N/A	N/A	N/A	6.36	6.05	5.74	5.58	5.57	5.65	5.40	1980-1985	1980-1985	2005-2010	1980-1985
Niger	N/A	N/A	N/A	7.97	7.97	7.97	7.97	7.97	7.97	7.81	7.16	1980-1985	Not found		1950-1955
Nigeria	7.20	7.20	7.20	7.17	6.93	6.66	6.38	6.11	6.02	5.68	5.38	1980-1985	Not found		1950-1955
Rwanda	N/A	N/A	N/A	8.04	7.74	7.09	6.01	5.99	5.70	5.37	4.81	1975-1980	1975-1980		1975-1980
Sao Tome and Principe	N/A	N/A	N/A	N/A	6.35	6.35	6.32	6.18	5.92	5.52	4.94	1980-1985	Not found		1950-1955
Senegal	N/A	N/A	N/A	7.29	7.18	6.80	6.36	5.88	5.47	5.12	4.69	1975-1980	1975-1980		1975-1980
Sierra Leone	N/A	N/A	6.50	6.48	6.44	6.40	6.20	5.76	5.32	5.07	4.90	1970-1975	1970-1975		1970-1975
Somalia	N/A	N/A	N/A	7.25	7.25	7.25	6.36	6.76	7.05	6.68	6.26	1975-1980	2000-2005		2000-2005
South Sudan	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	6.05	5.66	2005-2010	2000-2005	2005-2010	2005-2010
Tanzania	N/A	N/A	N/A	6.87	6.64	6.33	6.01	5.78	5.67	5.47	5.08	1975-1980	1975-1980		1975-1980
Togo	N/A	N/A	N/A	N/A	7.31	7.02	6.29	5.50	5.17	4.90	4.64	1980-1985	1980-1985		1980-1985
Uganda	N/A	7.55	7.55	7.55	7.54	7.45	7.32	7.18	6.96	6.55	6.14	1965-1970	Not found		1950 - 1955
Zambia	N/A	N/A	N/A	N/A	7.01	6.53	6.21	6.00	5.95	6.02	5.85	1980-1985	1980-1985	2005-2010	1980-1985

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